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	Published	Discussion closes
Eisenlohr, William S., Jr. Coefficients for Velocity Distribution in Open-Channel Flow	Jan., 1944	
Discussion in Apr., Sept., 1944		Closed*
Symposium: Military Airfields	Jan., 1944	
Discussion in Feb., Mar., Apr., May, June, Sept., Nov., Dec., 1944, Mar., 1945		Closed
Steinman, D. B. Rigidity and Aerodynamic Stability of Suspension Bridges	Nov., 1943	
Discussion in Feb., Mar., May, June, Sept., Dec., 1944		Closed*
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Discussion in Feb., Apr., May, 1944, Jan., 1945		Closed*
Wilson, David M. Analysis of Rigid Frames by Superposition	Feb., 1944	
Discussion in May, June, Sept., Nov., 1944, Mar., 1945		Closed
Gray, Hamilton. Simultaneous Consolidation of Contiguous Layers of Unlike Compressible Soils	Feb., 1944	
Discussion in June, Sept., 1944		Closed*
Komora, Andrew M. Erection of Hydraulic Turbines and Generators	Mar., 1944	
Discussion in June, Sept., 1944, Mar., 1945		Closed
Benscoter, Stanley U., and Logan, Samuel T. Shear and Bond Stresses in Reinforced Concrete	Mar., 1944	
Discussion in May, June, Sept., 1944		Closed*
Kinnison, H. B., and Colby, B. R. Flood Formulas Based on Drainage Basin Characteristics	Mar., 1944	
Discussion in June, Sept., Oct., Nov., 1944		Closed*
Amster, Maurice N. Hydraulic Model Investigation of Miter Gate Operation	Mar., 1944	
Discussion in June, Sept., 1944, Mar., 1945		Closed*
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Discussion in Sept., Oct., Nov., 1944		Closed*
Nubar, Yves. Analysis of the General Two-Dimensional Framework	Apr., 1944	
Discussion in June, Sept., 1944, Feb., 1945		Closed*
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Discussion in Sept., Dec., 1944		Closed*
Foster, H. Alden. Construction of the Flow Net for Hydraulic Design	May, 1944	
Discussion in Jan., Mar., 1945		Apr. 1, 1945
Vanoni, Vito A. Transportation of Suspended Sediment by Water	June, 1944	
Discussion in Dec., 1944, Feb., Mar., 1945		Apr. 1, 1945
Wilbur, John B. Characteristic Redundants Used for Analyzing Statically Indeterminate Structures	June, 1944	
Discussion in Sept., Dec., 1944, Jan., 1945		Apr. 1, 1945
Wilkinson, J. H. Translatory Waves in Natural Channels	June, 1944	
Discussion in Jan., Feb., 1945		Apr. 1, 1945
Moorman, Robert B. B. Wedge-Shaped Structural Members Under Direct Stress and Bending	June, 1944	
Discussion in Oct., 1944, Jan., Mar., 1945		Apr. 1, 1945
Terzaghi, Karl. Stability and Stiffness of Cellular Cofferdams	Sept., 1944	
Discussion in Dec., 1944, Feb., Mar., 1945		Apr. 1, 1945
Stewart, Ralph W. Amplified Slope Deflection	Sept., 1944	
Discussion in Nov., 1944, Feb., Mar., 1945		Apr. 1, 1945
Ahlf, August L. Design Constants for Beams with Nonsymmetrical Straight Haunches	Oct., 1944	Apr. 1, 1945
Soucek, Edward, and Zelnick, E. W. Lock Manifold Experiments	Oct., 1944	
Discussion in Mar., 1945		Apr. 1, 1945
Symposium: Concentration of Sewage Sludge	Oct., 1944	Apr. 1, 1945
Hickox, G. H. Evaporation from a Free Water Surface	Oct., 1944	
Discussion in Feb., Mar., 1945		Apr. 1, 1945
Bull, Anders. Stresses in the Linings of Shield-Driven Tunnels	Nov., 1944	
Discussion in Jan., Feb., 1945		Apr. 1, 1945
Johnson, Lee H., Jr. Analysis of Statically Indeterminate Structures Using Reduced Equations	Nov., 1944	
Discussion in Feb., 1945		Apr. 1, 1945
Powell, Ralph W. Flow in a Channel of Definite Roughness	Dec., 1944	May 1, 1945
Kalinske, A. A. Conversion of Kinetic to Potential Energy in Flow Expansions	Dec., 1944	
Discussion in Feb., 1945		May 1, 1945
Osgood, William R. Column Formulas	Dec., 1944	
Discussion in Feb., Mar., 1945		May 1, 1945
Blanks, R. F., and Meissner, H. S. Deterioration of Concrete Dams Due to Alkali-Aggregate Reaction	Jan., 1945	June 1, 1945
Bennett, Preston T. The Effect of Blankets on Seepage Through Pervious Foundations	Jan., 1945	
Discussion in Mar., 1945		June 1, 1945
Conkling, Harold. Utilization of Ground-Water Storage in Stream System Development	Jan., 1945	
Discussion in Mar., 1945		June 1, 1945
Weiss, Camillo. Interrelation of Certain Structural Concepts	Jan., 1945	June 1, 1945
Bakhmeteff, Boris A., and Allan, William. The Mechanism of Energy Loss in Fluid Friction	Feb., 1945	July 1, 1945
DeYoung, Isaac. A Plan for a Movable Dam	Feb., 1945	July 1, 1945
Report: Organization, Financing, and Administration of Sanitary Districts: Progress Report of the Committee of the Sanitary Engineering Division	Feb., 1945	July 1, 1945

NOTE.—The closing dates herein published are final except when names of prospective discussers are registered for special extension of time.

* Publication of closing discussion pending.

CONTENTS FOR MARCH, 1945

PAPERS

PAGE

Elimination of Cross Connections in Los Angeles, Calif.
By Harry Hayes 271

Economics of Photogrammetry.
By Leon T. Eliel 287

Determination of Pressures within a Siphon Spillway.
By I. M. Nelidov 297

Computing Coagulant Requirements in Sludge Conditioning.
By A. L. Genter 307

DISCUSSIONS

Erection of Hydraulic Turbines and Generators.
By Andrew M. Komora 331

Construction of the Flow Net for Hydraulic Design.
By Clarence S. Jarvis 333

Transportation of Suspended Sediment by Water.
By A. A. Kalinske 335

Hydraulic Model Investigation of Miter Gate Operation.
By William A. Tripp 338

Lock Manifold Experiments.
By J. Pietrkowski 341

Evaporation from a Free Water Surface.
By C. W. Thornthwaite 343

Column Formulas.
By Edward Godfrey 349

CONTENTS FOR MARCH, 1945 (*Continued*)

PAGE

Utilization of Ground-Water Storage in Stream System Development.
By Raymond A. Hill 351

Unusual Cutoff Problems—Dams of the Tennessee Valley Authority: A Symposium
By L. A. Schmidt, Jr. 358

Amplified Slope Deflection.
By William C. Spiker, Arthur B. McGee, and Jaroslav Polivka 361

Wedge-Shaped Structural Members Under Direct Stress and Bending.
By Walter H. Wheeler 366

Stability and Stiffness of Cellular Cofferdams.
By R. T. Colburn, A. F. Hedman, and Adolph J. Ackerman 367

Analysis of Rigid Frames by Superposition.
By David M. Wilson 394

The Effect of Blankets on Seepage Through Pervious Foundations.
By Reginald A. Barron 407

A list of "Current Papers and Discussions" may be found on the page preceding the table of contents

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AMERICAN SOCIETY OF CIVIL ENGINEERS

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PAPERS

ELIMINATION OF CROSS CONNECTIONS IN LOS ANGELES, CALIF.

BY HARRY HAYES,[1] ASSOC. M. AM. SOC. C. E.

SYNOPSIS

The City of Los Angeles, Calif., has undertaken what is probably one of the most carefully planned and executed programs for the control and elimination of cross connections of any city in the United States. This is the outcome of development work started by the City Department of Water and Power, at first acting alone but gradually acquiring the cooperation of all governmental units having jurisdiction, which act in unison through a joint committee. The purpose of this paper is to describe briefly some of the factors which enter into the development of this program, the formation of the joint committee, and results accomplished.

HISTORY

When potable water delivered to a consumer from a source approved for domestic consumption may be polluted or contaminated by admission of polluting material through a connection to the consumers' water piping system, such connection is known as a cross connection. (By the term "cross connection" as used in these regulations is meant any connection between any part of a water system used or intended to supply water for drinking purposes, and any other source or kind of water that is not or cannot be approved as reasonably safe and potable for human consumption whereby water from said unapproved source may be forced or drawn into the drinking water system.[2]) The existence of dangerous cross connections inimical to the public health has been recognized for many years. Publications are replete with reports of epidemics of water-borne diseases resulting in sickness and death which have been attributed directly to dangerous cross connections. Investigations indicate that the number of instances of sickness caused by contaminated drinking

NOTE.—Written comments are invited for immediate publication; to insure publication the last discussion should be submitted by **August 1, 1945.**

[1] Asst. San. Engr., Dept. of Water & Power, Los Angeles, Calif.

[2] "Cross Connection Regulations," adopted August 28, 1943, *Bulletin No. 64*, Dept. of Public Health, State of California.

water through backflow connections has been many times greater than the number actually reported in print. It is further apparent that, instead of diminishing, the number of instances where pollution has occurred is on the increase, due largely, it is believed, to the rapid increase and expansion of war industries. This is attested to by the reported incidents of salt-water pollution of drinking water lines in shipyards alone.

Many states have regulations governing cross connections, administered through public health authorities, some of which forbid the existence of cross connections even with secondary supplies that may be considered potable, whereas others allow such cross connections if the primary supply is protected by an approved double-check valve installation. As yet, no guaranteed procedure for backflow protection has been developed. Although some consider parallel piping systems with air-gap or broken connections as ideal, systems originally protected by such means have become cross connected by operators, which nullifies the supposed protection entirely. Others place reliance upon mechanical protective devices although such devices have not been fully developed to afford complete protection. It becomes a question of whether present mechanical deficiencies are more serious than the human element in making new cross connections. Current conditions in regard to cross-connection control by various state and local governmental bodies have been reported quite thoroughly by R. F. Goudey[3] and Robert E. Moore, Jr.[4]

The City of Los Angeles began conducting an increasingly vigorous campaign of cross-connection control and elimination in 1930, mostly through the Sanitary Engineering Division of the Department of Water and Power. Since that time most buildings in the downtown area, and many buildings in outlying areas, have been surveyed. Every new water-service sale is checked for use and all properties to which the services are sold, with the exception of dwellings with service connections less than 1.5 in. in diameter, are inspected for cross connections. In many cases properties or building plans are investigated before the service is sold, and, where it is found that backflow protection is required at the meter, the service is withheld until such protection is provided.

Until 1942, the control of cross connections on water piping systems connected to the municipal water supply mains was solely for the purpose of preventing backflow of polluted water into the street mains. That contamination of the water within a private water piping system might occur, with resulting danger to the health of occupants of the property, was not the primary concern of the Water Bureau so long as that contamination did not flow through the meter into the street main. Where contamination was known to have occurred, or was likely to occur, in a private water piping system, the matter was reported to the local health authorities for further action.

Right of entry on to private property for the purpose of investigating the water use, and for inspection for the existence of cross connections, was established by rules and regulations passed by the Board of Water and Power

[3] "Practical Aspects of Cross-Connection, Inter-Connection and Back-Flow Protection," by R. F. Goudey, *Journal*, A.W.W.A., March, 1941, p. 391.

[4] "Legal Aspects of Cross-Connection, Inter-Connection and Back-Flow Protection," by Robert E. Moore, Jr., *ibid.*, p. 495.

Commissioners[5] which have all the force and effect of general laws.[6] Other rules and regulations provide for the prohibition of cross connections on the consumers' property or protection of the water supply and the right to discontinue water service for violations.[7]

On a routine investigation of a water service for cross connections the inspector began at the meter and followed the water supply piping from the point of entry to the property, through all branches to the last foot of pipe, inspecting every connection, outlet, and water-using device and fixture found on the service. Every cross connection was listed and located, using sketches and drawings where required. When the survey was completed, the cross connections were listed (except cross connections to plumbing fixtures which were referred to the Department of Building and Safety) and presented to the owner or person in charge of the property with a legal order requiring their elimination from the potable water piping system.

Two alternates were allowed: The first was the complete severance of each individual cross connection on the system; and the second was the installation of a backflow protective device at the meter. By the use of the first method, the public water supply system was protected so long as no new cross connections were made. By the use of the second method, the mains were reasonably well protected so long as the checks were maintained in good condition. In either case, direct connections to the sewer were ordered severed.

Organization and Functions of the Committee for Cross-Connection Control

In 1942 the Cross-Connection Control Advisory Committee was founded to take over cross-connection control and elimination for the entire city. It was formed by a joint resolution passed by the Boards of Commissioners of the Building and Safety Department, the Health Department, the Water and Power Department, and the Board of Mechanical Engineers of the City of Los Angeles. Its membership is comprised of one representative from each of the departments and one attorney from the city attorney's office, plus certain advisory counselors who may be appointed from time to time. The sanitary engineer of the Department of Water and Power is the member from that department and is the permanent chairman of the committee.

The committee is a quasi-legal organization and is invested with all the police powers of the signatories to the joint resolution, in so far as they pertain to cross-connection control. The policy of the committee is to accomplish its objectives through education and cooperation rather than through enforcement, using its authority and the laws back of it only as a last resort. The committee has sponsored evening classes in Los Angeles at the University of Southern California and the Frank Wiggins Trade School for the purpose of educating the public on what constitute cross connections, the dangers of allowing their existence, and the methods for correcting or eliminating them.

[5] *Rule and Regulation No. 58*, Board of Water and Power Commrs., City of Los Angeles, Calif.

[6] *City Charter*, City of Los Angeles, Calif., Chapter I, Sections 82 and 220.

[7] *Rule and Regulation No. 59 to No. 62*, inclusive, Board of Water and Power Commrs., City of Los Angeles, Calif.

thus protecting the water supply system. These classes have been well attended by plant engineers, master plumbers and journeyman plumbers, plumbing and health inspectors, representatives of manufacturers of plumbing equipment, and many others who are concerned with the maintenance of the purity of the water supply system.

Field inspection forces were obtained by assigning to the committee inspectors from the various departments represented. A chief inspector and two assistants, with previous experience in this type of work, were selected from the engineering staff of the Sanitary Engineering Division of the Department of Water and Power to take charge of the inspection work. Most of the inspectors who have worked under the jurisdiction of the committee were obtained from the Plumbing Inspection Division of the Department of Building and Safety because the type of work involved is akin to plumbing inspection.

The procedure in making a survey under the authority of the committee is identical to that used by the Department of Water and Power before the committee was conceived. Great care is taken to see that each pipe line is traced to its end and every cross connection located. When the survey of a property is completed, the inspector writes a legal order requiring the elimination of all cross connections found having listed them in an attached appendix. In making up the list each cross connection is numbered consecutively, its location given, and the method of elimination stated. Thereafter it is referred to by number only. Wherever possible, cross connections of a like nature that can be eliminated by similar methods, such as the raising of low inlets into tanks, or installation of vacuum breakers on flush valves, are grouped together so that the order for elimination covers the entire group.

For many items on the list there is but one method of elimination. If such is the case, it is so ordered, but for others where the elimination may be accomplished by any of a number of methods, involving, perhaps, complete changes in piping systems, the owner is given the option of using the method that serves his purpose best.

In order to further unify the methods of elimination for all properties so that none will be discriminated against or given special advantages, a code of standard wording was devised for each type of cross connection, and in writing up his orders each inspector must follow this standard explicitly. The only time when deviation from the standard is allowed is when a new type of cross connection or an unusual condition is found that is not covered by the standard wording, and in that case it is referred to the engineers for codifying.

The legal order is dated as of the day it is served; and the person to whom it is served, or his agent or representative, is required to sign the order to acknowledge receipt of it. The order is then dated as to the time allowed for completion of the work ordered or provision made for its completion. The time limit given is usually two weeks. If the required work cannot be completed in this time, the owner is allowed to file biweekly progress reports and the limit placed on the time of completion is extended as long as evidence of good faith is shown. However, no order is allowed to be pigeonholed and forgotten.

Each inspector working on cross-connection control work, regardless of the department from which he is obtained, is deputized to act for each of the

member departments in cross-connection control work. His duty is not only to insure the purity of the water supply in the city mains, but he is also interested in protecting the public health by the elimination of all cross connections and illegal plumbing installations on private premises that might adversely affect the purity of the water in the consumers' water piping system and endanger the health of any individual drinking from that system.

The three engineers from the Department of Water and Power and two plumbing inspectors from the Building and Safety Department who had had some experience in cross-connection inspection work formed the nucleus for the inspection. From time to time inspectors from other member departments were added. Each new man, when assigned to the field inspection force, was paired with an experienced man for a period of training until he was able to do the field work alone and perhaps train new men. The purpose of the committee was to allow every inspector working for a member department sufficient time on cross-connection inspections to become thoroughly familiar with the work so that he could recognize and order the elimination of any cross connections he might find when conducting his regular department inspection work. The time required to train regular plumbing inspectors in this work depends on the ability of the men assigned. Exceptional men will require from six to nine months; men who are slower to learn will require a year or more; and some will never be able to make a satisfactory survey. Overconfidence on the part of the trainee is the greatest stumbling block to overcome. It takes at least thirty days on the job before a conscientious inspector discovers how little he knows.

Inspections of water piping systems in buildings in the business district, food processing plants, hotels, hospitals and sanitariums, manufacturing plants, etc., that had previously been conducted by the Water Department engineers were undertaken by the committee's inspection force. With the outbreak of World War II and the necessity for a continuous flow of war materials, the committee confined its inspections more and more to plants and industries connected with the war effort. This safeguarded large concentrations of war workers and it was also found that preferential ratings could be used for obtaining the necessary critical materials. The inspection of residential, business, and other properties not connected with the war effort was abandoned for the duration, with the exception of special cases involving sickness.

Evidences of Pollution

Early in 1942, a series of incidents occurred which focused the attention of the committee on the Los Angeles Harbor area. The first of these was a report from the Sanitary Engineering Division of the Water Department that analyses of water samples collected at certain locations from the water distribution system in the harbor area showed evidences that pollution was getting into the mains from some source in the area. A sanitary survey indicated the possible sources of pollution as a shipbuilding yard and an army fort. The bacterial pollution appeared to be coincident with periods of heavy water use in the shipyard which caused low water-pressure conditions in the street mains and distribution systems serving both the yard and the fort. A quick check

up showed both places to be replete with cross connections. An order was issued immediately that backflow protection be installed at the meters on all services entering the suspected properties. Thereafter there appeared to be less evidence of pollution in water samples collected from the city water mains in that area, but the water in the mains in the fort continued to show evidence of contamination and was quarantined for drinking purposes.

Systematic and complete cross-connection surveys of each of the properties revealed the existence of more than 350 cross connections in the shipyard and nearly a thousand at the fort.

Second in the series of incidents was the actual pollution of the fresh drinking water lines within the aforementioned shipyard. A ship's officer on board a ship tied to a wharf connected the fresh-water system in the yard to the salt-water system on board the ship, started the ship's pumps, and pumped sewage-polluted salt water throughout the yard's drinking water system. This was the first of five occurrences within six months where polluted water was forced back into drinking water systems in the harbor area, and although there are no official records of sickness resulting from these occurrences, it is known unofficially that there were many cases of illness due to these causes, and the loss of man-hours in the plants involved was high.

Such occurrences have not been confined to plants in Los Angeles or in California. Reports have been published of similar cases occurring in shipyards and industrial plants in various parts of the United States, indicating that hazardous conditions exist in all hastily built war production plants because, in most cases, it is impossible to get designers and engineers who have a practical as well as a theoretical knowledge of cross-connection control.

One of the most hazardous conditions found by the committee was the existence of many cross connections between the fresh-water domestic and fire-fighting systems and secondary piping systems carrying sewage-polluted salt water. Most of these were newly made and were intended to augment the supply of fresh water for fire-fighting purposes and many involved new installations of auxiliary salt-water pumps.

The committee notified the State Board of Public Health of the conditions found in the harbor area, with the request that action be taken to assist in removing the dangers. As a result, that Board called a meeting on June 22, 1942, of all city and county health officials of Southern California and other interested parties, for the purpose of considering this problem. As an outcome of this meeting the State Board of Health, on June 27, 1942, passed a resolution[8] condemning the use of cross connections and requiring their elimination or control. This same resolution was adopted by reference by the Los Angeles City Board of Health and became a law in the city on October 8, 1942.

The Seven-Point Program

In August a meeting was called by the committee and was attended by representatives of various federal, state, and local governmental agencies to

[8] *Resolution Relative to the Protection of Water Supplies Against Pollution Due to Cross-Connections with Auxiliary or Emergency or Firefighting Supplies*, State Board of Health, State of California, adopted June 27, 1942.

discuss plans and procedures for cleaning up the cross-connection situation in war industry plants in Los Angeles, in general, and in the harbor area in particular. It was agreed that the committee should undertake this job and that ten inspectors from the Building and Safety Department would be assigned permanently to the committee for this work.

On September 15, 1942, a meeting of representatives from numerous shipyards and industrial plants, the U. S. Army, Navy, Coast Guard, Maritime Commission, the U. S. Public Health Service, State Department of Health, City Council, and other governmental departments concerned was called by the committee at the U. S. Naval Operating Base at Terminal Island (California) for the purpose of acquainting the representatives with the committee's proposed plan of survey and the policies to be followed by the committee in requiring cross-connection control and elimination and to discuss backflow protective installations.

The following seven-point program was presented to and adopted by the group:

1. Backflow protection at meter of all services to premises fronting on the harbor and to adjacent industrial and commercial plants.
2. Backflow protection at the wharf for all ship supplies.
3. Segregation of domestic and fire lines as far as possible so that no unprotected cross connections will endanger the domestic supply from pierhead, Siamese, or salt-water pump intakes.
4. Elimination of all dangerous plumbing cross connections.
5. Provision for the isolation of all pipe lines carrying processing water.
6. Adoption of color and sign scheme for designation of safe and unsafe water lines.
7. Organization of water foremen on each premise to supervise a unified plan.

These seven points represent the uniform policy established at the meeting of September 15 and followed by the committee.

On September 10, 1942, an office was opened in Wilmington, Calif., with ten inspectors and three engineers assigned to the survey. Work was started immediately on the waterfront properties. Preliminary contacts with plant managers were made by the engineers to arrange passes and entry for the inspectors, to obtain maps and drawings of the water piping system on the property if available, and to enlist the cooperation of the plant engineer in making the survey. In general, the cooperation of the management of the plants has been very satisfactory, but, as with individuals in a group, there are contrary ones who make it difficult for the inspectors. Although the inspectors carry sufficient legal authority to enter any of the plants for inspection purposes, it is the policy of the committee never to make use of such authority or to make any show of force in dealing with the plants unless compelled by a refusal of cooperation. That holds true also in the issuance of orders for the elimination of cross connections or any other required work for the protection of the drinking water system.

The survey proceeded with diligence, and the report to the committee (dated January 1, 1945) shows that the following had been accomplished as of that date:

Premises surveyed and legal orders issued	267
Cross connections found	12,500
Work completed on orders issued	99%
Survey of harbor area completed (industrial area only)	100%
Approximate cost of work ordered	$600,000

As the report shows, the survey of the harbor area is completed. Part of the inspector's duty is to be available on call to consult with plant managers in perfecting methods for the required elimination. As the work in the harbor is completed the inspectors are moved north into defense plants in the "shoestring" strip connecting the harbor area and the city and into the city area.

During 1943, the District Security Officers for the Army, Navy, Air Force, and other branches of the armed services were charged with the responsibility of securing the safety of the drinking water systems in all plants on war material production under their jurisdiction. As the Cross-Connection Control Advisory Committee was at the time proceeding with its program of cross-connection elimination in war production plants, it was requested by the District Security Officers to expand its activities to include water safety security.

The committee was supplied with master responsibility lists of war production plants, and surveys were made at these plants in accordance with their priority on the lists.

When the survey of a plant was completed, and an order for corrections was issued, a copy of the order was forwarded to the District Security Officer, who thereupon assumed responsibility for its enforcement. The committee, however, maintained a continuing inspection at the plant to see that the terms of the order were fulfilled.

(*1*) *Protection at the Meter.*—The requirement of backflow protection at the meter (point 1) is not confined to harbor-front properties alone, but to any plant or property having access to auxiliary water supplies and pressure producing equipment for utilizing such supplies. Fig. 1 comprises two reproductions of prints issued to property owners with orders for cross-connection elimination involving such necessary protection, showing various methods which have been approved by the committee, for installation at the meter, to protect against the pollution of the water supply in the street main.

The fundamental principle of backflow protection is the air-gap or broken connection. There is as yet no completely satisfactory substitute for a broken connection or a complete severance of a cross connection for permanent protection to a drinking water system. However, modern backflow protective equipment has been developed that will give a reasonable degree of protection if properly installed and maintained.

The first method of protection shown in Drawing No. A-1, Fig. 1, makes use of the air-gap principle by requiring the water supply to be discharged

into a receiving tank through an overhead inlet which is sufficiently high above the overflow rim of the tank to prevent any siphoning back of water from the tank should a vacuum occur in the supply main. Furthermore, it is obvious that any pressure producing device in the consumer's water piping system could not force water back across this gap (as would be possible through a direct connection), either against the pressure in the main or into a vacuum that might occur in the main.

In general, the least air-gap distance required is twice the diameter of the inlet pipe, plus additional distances in special cases. As shown in Drawing No. A-1, the tank may be placed in an elevated position so as to provide gravity pressure in the consumer's water piping system, or the pressure may be produced by a pressure pump taking the suction out of the receiving tank. A device not shown in the drawings, which is the nearest equivalent to an air gap in protection, and which allows a solid connection to the water supply so that the main pressure can be utilized, is the siphon loop. This is a loop in the supply pipe which rises to a height of not less than 35 ft above the highest point from which back-siphonage of polluting material might occur. It can be used only on premises where there are no steam boilers, auxiliary water supply, or any pressure producing device connected into the water piping system except where the loop is raised to a height sufficient to compensate for the maximum pressure that may occur.

It is seldom practicable to use such a loop, as it requires a support that may extend a considerable distance above the roof of the building housing the piping system, although it has been recommended at times where conditions seem to warrant its use.

Where it is impossible or impracticable to require the use of the foregoing methods of protection on a water service, the committee allows the use of backflow protective devices of approved types, provided they are installed in an approved manner and maintained in good operative condition at all times. Such approved devices are shown in Drawing No. A-1, alternate method (Fig. 1) for services less than 6 in. in diameter, and Drawing No. A-2, Fig. 1, a typical double check-valve installation for 6-in. services and larger. Any device for such use must be submitted to the laboratory of the Building and Safety Department for tests and approval before its use is allowed in Los Angeles.

Although it is recognized that no mechanical device is the equivalent of a broken connection for backflow protection, great progress has been made in the development and the design of modern backflow protective devices. The trend is toward the perfection of the operating mechanism together with the use of devices which operate to destroy the negative and positive forces that cause backflow.

In general, the forces that tend to cause backflow through any cross connection are: (1) Reduced pressure or a vacuum in the water supply mains caused by main breaks, shutdowns for repairs, excessive water use, pumping connections for fire fighting, etc.; (2) gravity head of water in the consumer's piping system; (3) pressure producing equipment connected into the consumer's

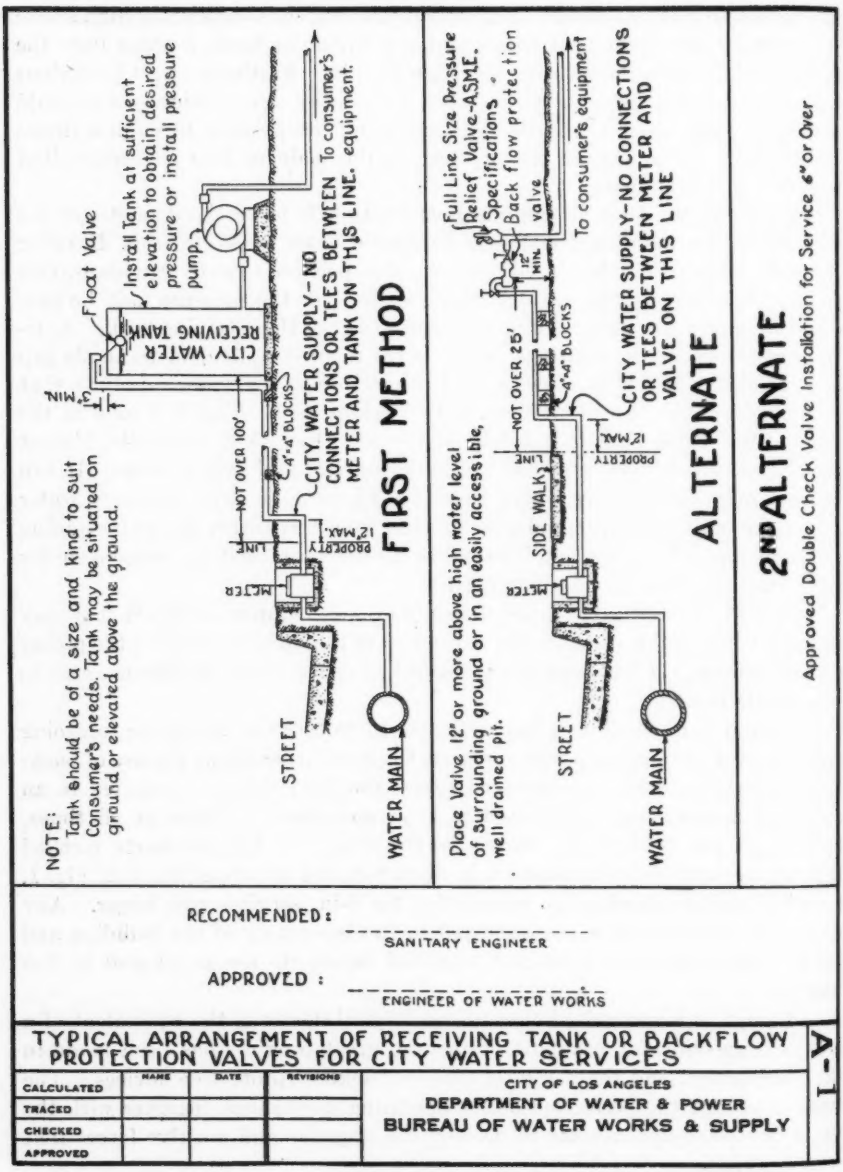
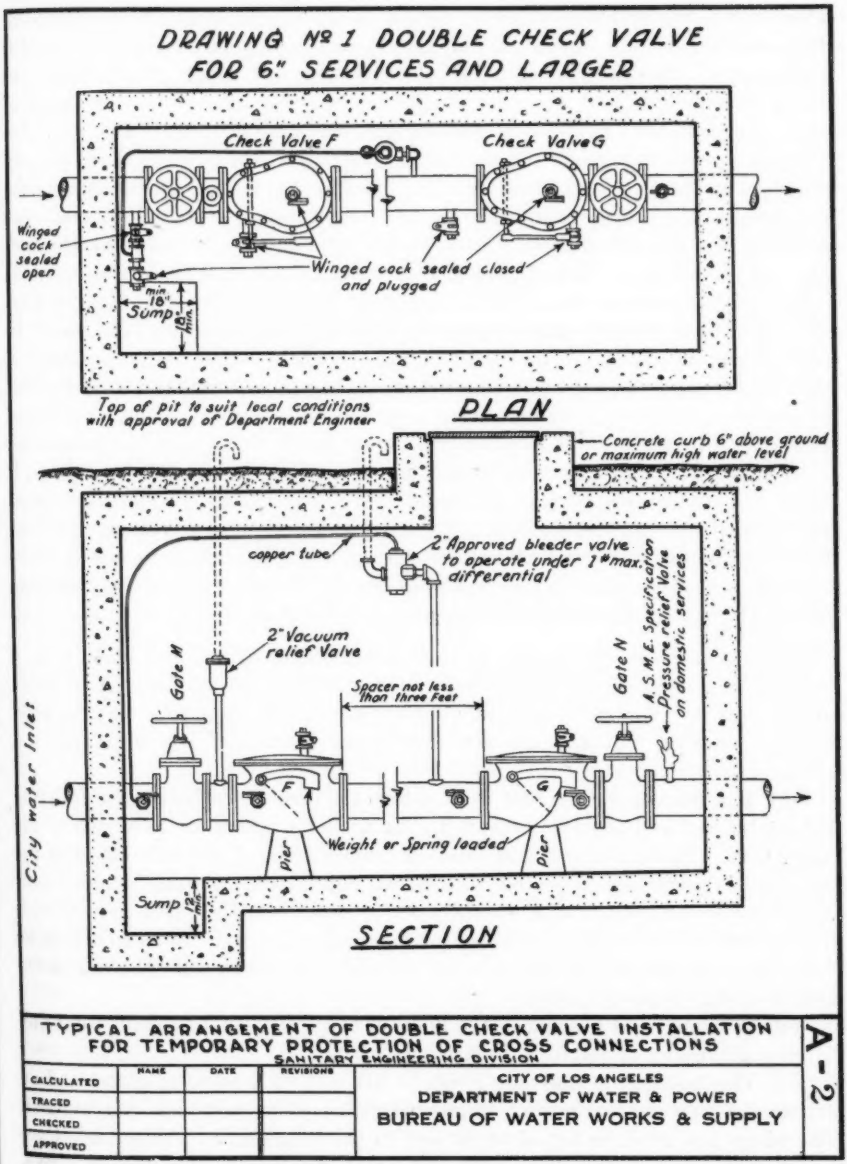

NOTE:
Tank should be of a size and kind to suit Consumer's needs. Tank may be situated on ground or elevated above the ground.
3' MIN.
Float Valve
Install Tank at sufficient elevation to obtain desired pressure or install pressure pump.
CITY WATER RECEIVING TANK
NOT OVER 100'
4"-4" BLOCKS
CITY WATER SUPPLY - NO CONNECTIONS OR TEES BETWEEN METER AND TANK ON THIS LINE.
To consumer's equipment.
FIRST METHOD
LINE
PROPERTY
12" MAX.
METER
STREET
WATER MAIN
Place Valve 12" or more above high water level of surrounding ground or in an easily accessible, well drained pit.
Full Line Size Pressure Relief Valve-ASME. Specifications
Back flow protection valve
12" MIN.
NOT OVER 25'
SIDE WALK
To consumer's equipment
CITY WATER SUPPLY-NO CONNECTIONS OR TEES BETWEEN METER AND VALVE ON THIS LINE
ALTERNATE
2ND ALTERNATE
Approved Double Check Valve Installation for Service 6" or Over
RECOMMENDED :
SANITARY ENGINEER
APPROVED :
ENGINEER OF WATER WORKS
TYPICAL ARRANGEMENT OF RECEIVING TANK OR BACKFLOW PROTECTION VALVES FOR CITY WATER SERVICES
A-1
CALCULATED
TRACED
CHECKED
APPROVED
NAME
DATE
REVISIONS
CITY OF LOS ANGELES
DEPARTMENT OF WATER & POWER
BUREAU OF WATER WORKS & SUPPLY

FIG. 1.—EXAMPLES OF PRINTS ISSUED TO PROPERTY OWNERS

water piping system, or connections to a secondary water piping system having a higher (or occasionally higher) pressure than the primary system.

The fundamental principle of all backflow protective devices for installation



WITH ORDERS FOR THE ELIMINATION OF CROSS CONNECTIONS

at the meter has been the use of double check valves. Until recently they did not include vacuum breakers and pressure-differential relief valves which have been included to destroy the forces creating backflow. Use of the double

check valve was first recommended by the New England Water Works Association, and has been adopted without question by many state and local health authorities. The original recommendations were for check valves made entirely of bronze, but, as the cost is high for the larger sizes, the Department of Water and Power, in some of the early installations, allowed the use of a certain type of checks made of cast iron with galvanized body which has proved quite satisfactory. These checks were originally developed as detector checks on fire-sprinkler services and for compound meters where it was found desirable to check the main flow-through channel in order to force small flows through the by-pass meter. To accomplish this purpose the clapper of the check is weighted internally to cause positive closure of the valve under static or low-flow conditions and to require a substantial decrease in pressure on the property side to make it open. In addition, the valve contains other desirable features, such as bronze seat rings, hinge and hinge pins, rubber or composition bushed hinge-pin bearings, and rubber-faced clapper disk. This type of check valve is the first choice of the committee for use in double check-valve installations.

With the increased scarcity of strategic materials due to war conditions, it has become increasingly difficult to obtain valves meeting previous specifications and the committee has allowed the use of other types of valves, provided they are rebuilt to meet certain requirements. Practically any check valve that has a bronze seat ring and hinge-pin bearings in the body of the valve can be rebuilt to meet the requirements, which are as follows:

(*a*) The face of the clapper disk must be machined flat and fitted with a rubber face composed of fabric-reinforced medium soft rubber. Any valve already on the market which has a rubber face in the seat ring may be used without change; but the clapper disk must be bronze, or must have a bronze face.

(*b*) The bearing for the bolt that fastens the disk to the hinge arm must be rubber bushed and must be provided with rubber washers for each side of the hinge arm. These washers and bushing must provide a tight fit between the disk and hinge arm and yet be resilient enough to allow sufficient movement for proper seating of the disk when the valve closes. If the valve to be used is double-hinged with rubber bushings for the hinge pins, or, if it has an otherwise solid but resilient connection between the disk and hinge arm, the aforementioned change is not required.

(*c*) The hinge-pin bearing in the body of the valve must be drilled out and fitted with rubber or reinforced plastic bushings.

(*d*) The bearing on one side must be fitted with a packing gland and an extended hinge pin of bronze, stainless steel, or other rust-resisting material. The hinge pin must be keyed or pinned to the hinge arm. An outside lever must be fastened to the hinge pin with a key, or a substantial pin, in such a position that it is horizontal when the valve is closed.

(*e*) A weight must be attached to the arm lever and adjusted in a position that will require a differential pressure of approximately 1 lb per sq in. to open the valve. Check valves are available which are already fitted with a packing gland and outside weighted lever arms as required in items (*d*) and (*e*).

The committee made a study of double check-valve installations and recognized the fact that double check valves alone cannot be relied upon to provide adequate protection against backflow. Therefore, the installation of other devices was specified in conjunction with the check valves for the purpose of dissipating the forces that cause backflow. Referring to Drawing No. A-2, Fig. 1, it will be seen that there is a vacuum-relief valve inserted in the water supply line ahead of the first check valve. Whenever a vacuum occurs in the city's water supply line, this valve will open to supply air to satisfy the vacuum, thereby reducing the negative pressure on the inlet side of the check valve.

Again referring to Drawing No. A-2, it will be seen that a differential pressure-relief bleeder valve is connected into the spacer between the two check valves with a control line connected into the main supply line ahead of the first shutoff gate. This valve (which is power operated by water pressure) opens to atmosphere when the water pressure in the main drops below the pressure in the spacer between the checks, thus allowing the excess pressure to bleed off. This valve is set to operate on approximately a 1-lb differential but its action must be retarded so as not to be affected by water hammer.

The purpose of these two devices is to maintain an equalized pressure on both sides of the first check valve so that there will be no force to cause backflow through that valve if it leaks or is defective. Backflow protective devices for smaller services must incorporate these principles and in addition must have three checks, or their equivalents.

The latest development, in equipment for protection against backflow, is a device incorporating a zone of reduced pressure in the line of flow through the device. The chamber between the main barriers to backflow is maintained at a pressure lower than the inlet pressure for all conditions of flow, by means of weighted, spring-loaded, or power-operated check valves, and the differential pressure thus created is utilized to control the operation of a port opening to atmosphere from this central chamber.

The operation of this port has a dual purpose. It will operate to maintain a reduced pressure in the central chamber if leakage occurs into the chamber through failure of first barriers under normal conditions, or through the second barrier under backflow conditions. A small amount of bleeding from the central chamber occurring at intervals during extremes of fluctuation of pressures in the supply main is normal to its operation, and serves to maintain its automatic features in good operating condition. Excessive bleeding, however, indicates failure of the main barriers and the need for servicing.

(2) *Protection at the Wharf.*—All pierhead outlets, on wharves, for supplying fresh water to ships must be protected against backflow (point 2). In every case this means that a protective device must be installed on each individual outlet as it is possible for a ship, tied to a wharf with fresh-water lines from the pierhead outlet aboard, to pump sewage-polluted salt water not only into the supply mains but into the fresh-water tanks of all other ships taking on water at that wharf or into their own. The city is legally responsible for the delivery of fresh water to all ships taking their supplies from its mains and cannot tolerate the pollution of the water supply of one ship by another through

water lines being fed from the city mains. These indicants are of common occurrence and have been the cause of much sickness, inconvenience, and loss of man-hours in shipyards all over the United States.

(*3*) *Pollution Through Fire Lines.*—Domestic water lines and fire-protection lines must be separated as far as possible where there are auxiliary pumps or intakes, or cross connections to a polluted secondary water supply (point 3).

In a large percentage of the plants in Los Angeles the domestic water lines and the fire-sprinkler lines are entirely separate systems using the same source of supply and are required to be maintained separately by the Rules and Regulations of the Department of Water and Power. Backflow protection is required on these fire-fighting lines only where there is equipment or conditions that may cause backflow of polluted water into the city water mains.

In many plants a single system is used for domestic and fire-fighting purposes and is supplied from the city water mains. Such arrangements have been called combined systems. Backflow protection on these systems is required only where local conditions show the necessity for it. In many large plants using a combined water system, and particularly in the Harbor area, auxiliary fire-fighting water pumps with polluted sources of supply (such as salt water) have been connected into the combined systems. These pumps in most cases are inoperative at all times except in case of fire and a dire need for water. In these plants the committee allows the cross connections to be maintained provided a permanent automatic chlorinator, delivering chlorine gas, is connected into the pump discharge or into the lines delivering the auxiliary water supply. The chlorinator must be maintained in a continuously operative condition so that, whenever the auxiliary pumps begin to operate and polluted water starts to flow into the combined water lines, it will open up full force and thoroughly chlorinate all water going into the system. At the end of the emergency operating period the combined system must be thoroughly flushed out with heavily chlorinated fresh water. A fire alarm must be in operation when the polluted water pump is operating; and a water foreman must be in constant attendance to report each instance.

(*4*) *Plumbing Cross Connections.*—The requirements that all plumbing cross connections must be eliminated (point 4) are covered by plumbing ordinances and include such specifications as low inlets to tanks and basins, toilet flush valves without vacuum breakers, and other items which can, and have, caused extremely hazardous conditions where the water piping system has been overloaded. The need for proper design of water piping systems and the elimination of plumbing hazards have been largely ignored by war plant contractors, under the erroneous idea that they were constructing a federal government operated or contractual plant and therefore were above all local ordinances and regulations. These items have been cleared up in the survey.

(*5*) *Pollution Through Processing Water.*—In many plants the drinking water lines are used for supplying water for industrial or processing purposes and are usually connected directly into the processing equipment so as to utilize the main pressure (point 5). In ordinary times when material is available the committee would require a separate system for carrying all industrial

or processing water. With the present scarcity of material, however, it is the policy of the committee to allow the use of the domestic water for supplying industrial processes, provided the lines running to the processing equipment are "industrialized" or isolated from drinking water systems by the installation of backflow protective devices at the point of take off, so that, under all conditions of use, water cannot be forced or siphoned back into the drinking water lines. Such corrections may require the running of a small-size line to supply drinking fountain, lavatories, etc., but, in general, such changes will call for the use of the minimum amount of material that will give the maximum amount of protection.

(6) *Color Warnings.*—Wherever there are two water systems in a plant, one carrying a safe, potable water used for drinking and other domestic purposes, and the other carrying a polluted, unsafe water (or water which is not protected and might at any time become polluted and unsafe), the committee requires that both systems be painted a distinctive color (point 6) so that plumbers or pipe fitters will not by accident make a cross connection between the two lines. In addition the committee requires that all outlets on the secondary system be posted with a reasonably large placard signifying that the water from that outlet is dangerous and should not be used for drinking purposes. The colors recommended by the committee for this use are solid blue, or blue stripes, for the drinking water lines, and red for the polluted lines. On the waterfront, maritime or naval standards may be used and are also recommended.

Proctorship Supervision.—An organization of water foremen for each premise, to supervise a unified plan of water protection, was started early in the program (point 7). The plan was to select, or have selected, from each plant or premise surveyed, one man or several men who would be thoroughly familiar with, or who would make themselves familiar with, the entire water piping system on the premises where they were employed. The plant engineer, or one of his pipe foremen, would be the logical man for the post. These men were to be made responsible for the purity of the water on the premises and were to see that no new cross connections were made on the premises, that all new installations of water piping systems were in accordance with existing laws and regulations, and that in those plants governed by point 3, where emergency fire-fighting connections are allowed to exist, all the precautions for maintaining the purity of the drinking water were observed. This organization has proved quite satisfactory, and has since been made a requirement in all plants by the State Board of Health.[2]

Summary

Approximately 50,000 cross connections had been eliminated in Los Angeles by December 1, 1944, representing sizes from 1 in. to 12 in. in diameter at a total cost well over $3,000,000 to the consumer. Since the war this program has embraced practically all the defense plants and military establishments in the Harbor area and many such installations in other parts of the city. It is gratifying to note that all local Army, Navy, Coast Guard, and defense plant

organizations are supporting this program. The War Production Board has approved the use of critical materials as required.

The committee is cognizant of the fact that the situation in Los Angeles is not unique but that conditions are duplicated in every war industry center. By adhering to its cross-connection control and elimination program, the committee hopes to make the city the safest place for industrial and other workers in the United States.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

PAPERS

ECONOMICS OF PHOTOGRAMMETRY

BY LEON T. ELIEL,[1] ESQ.

SYNOPSIS

Contour mapping is rapidly becoming mechanized by the application of photogrammetric methods. Mosaic maps are widely accepted today. Planimetric maps are made and used extensively by many government agencies. There are a variety of photogrammetric processes which are quite controversial. The economics of the various photogrammetric methods and plane table surveys show why the different methods are used.

GENERAL

"Aerial photography" encompasses everything in the entire process of taking pictures from the air. When the original picture itself is all that is desired, the work pertaining to it lies entirely in the realm of aerial photography. When this picture is to be transformed into an instrument of precision, however, such as a precise mosaic, a planimetric or contour map, the work is known as "photogrammetry." Fundamentally the word means: "Picture measuring."

Photogrammetry in the United States dates from the early 1920's. In Europe it was practiced prior to World War I, and from the Europeans and, in fact, alas, from the Germans, the United States received most of its original knowledge of photogrammetry.

Only a few years ago most engineers were being disillusioned after their introduction to aerial photography. They had been led to believe that the aerial photograph, which was then procurable, was, in fact, the equivalent of a map. How far from the truth this was they soon found out. They discovered that the picture had certain errors and distortions. They found that the scale of the picture varied with differences in the elevation of the ground. Pictures were assumed distorted near the edges, although the center was known to be reasonably free from these aberrations.

These early undesirable characteristics of the aerial photograph proved

NOTE.—Written comments are invited for immediate publication; to insure publication the last discussion should be submitted by **August 1, 1945.**

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to be a blessing in disguise. The distortion is geometric, of course, and the picture is nothing but a conic projection. Thus points are located along radials from the principal point or center of the picture, in accordance with their elevations. Consequently, a vertical object, such as a telegraph pole, an oil derrick, or the Washington Monument, photographs as a radial image, the top being radially displaced in relation to the base.

In fact, it is this so-called parallax of the picture which makes possible the viewing of an over-lapping pair of vertical photographs stereoscopically. This same parallax can be measured to determine differences in elevation from a stereoscopic pair of pictures, and these geometric qualities are the foundation of all photogrammetric systems.

Mosaic Maps Have Arrived

The mosaic map of the early days was the work of an artist dedicated to the principle that no error was serious unless it could be seen. Thus, these early mosaicists were in reality a breed of prestidigitators—so clever that they made things look all right even when they were all wrong.

Today, cleansed by the blessing of a scientific approach, the mosaic has been accepted in practically all circles as a good map for many purposes, and good maps have been needed, especially by the U. S. Army. Only about 25% of the United States was adequately mapped prior to World War II. Since that time the precise mosaic, accurate and quickly made, has become a map acceptable for many military purposes.

Mosaics are built on a graphical triangulation net known as radial control, which depends upon the fact that all displacements due to difference in ground elevation are radial, and consequently the angle subtended at the center of the picture by any two points is approximately a true angle. Thus, by using merely the direction of various image points from the center of the picture, rather than the images themselves, as a triangulation net, it is possible from the pictures themselves to establish the true position of any number of points in each picture. An occasional point of known position establishes the desired scale and the necessary geodetic ties. The true position of various points in a picture can now be compared with the image position, and a computation made by which the picture may be brought to scale. It is enlarged or reduced, and, in case of sloping ground, tilted so that the scale of one side is corrected more than the other. In mountainous terrain, pyramiding is utilized, whereby the picture is broken up into small sections, each of which can be brought to a uniform scale so that the picture can be laid accurately to the radial control.

Slotted Templet Method of Control.—One widely used method of radial control is known as the "slotted templet method." By this method a templet, usually cardboard, is made from each individual photograph. The principal point, or center point, of the photograph is pricked through from the picture to the cardboard, and at least eight radial points around the edges are similarly designated. A hole is punched in the center of the picture, and through each of the outlying points a radial slot is cut. Due to the overlap of the pictures, each of these points is common to several pictures—sometimes as many as

six. The templets are then "buttoned up" with connecting studs through the slots, so that all the slots representing a given common point from the various pictures are assembled over a stud. Thus an entire map can be "buttoned up" and, due to the freedom of the slots, can be enlarged or reduced accordion-like, so that the entire area automatically conforms to a given scale. A very few control points, established by conventional surveying, serve to establish the map scale, and the map stretches or shrinks with movement in its radial slots to conform to these pre-established points.

When the entire area is covered by the templets, each one of which represents one of the pictures, the studs that hold the templets together represent the true position of an infinite number of myriad points. These studs have a hole through which a pin can be applied to the board beneath the templets. The points are pricked through each stud and properly designated, thus becoming the field notes, as it were, from which the ultimate map can be compiled. Typical mosaic specifications may require that 90% of all points shall be within 0.05 in. of their true location.

Planimetric Maps

Planimetric maps are compiled in much the same way as mosaics. Ordinarily, the first step in the compilation of a planimetric map is to make the radial control, or slotted templet, base. This base is traced on to tracing cloth or acetate sheets, and the planimetric detail is added by projecting the photographs on to the sheets in such a way that the control points on the photographs conform with their map positions as established on the radial control plot. Government agencies, such as the U. S. Geological Survey and the Soil Conservation Service, have done considerable planimetric mapping since about 1936.

Mechanized Contour Maps

Whereas the main problem in constructing mosaic and planimetric maps is to eliminate the effects of parallax, the contour map, on the other hand, is made by measuring this parallax. The aerial photograph, when properly taken, is a perfect triangulation record, made as though a transit were set at the same point in space as the lens of the camera. Two such pictures taken from the extremities of the air-base line afford the means by which all of their common points may be located by photogrammetry which, in common language, is triangulation by photography. To be sure, the modern photogrammetric machine does its work automatically, so that it is unnecessary to solve triangles for each individual point. However, were one inclined to this tedious process, a perfectly satisfactory answer could be derived in this way.

Two-Dimensional Processes

The Stereocomparagraph.—Contour drawing machines are essentially of two types. The first is the parallax measuring stereoscope, which is a two-dimensional machine, drawing its contours to the same conic projection as the picture itself. The most widely known machine of this type is the stereo-

comparagraph (not to be confused with the stereoplanigraph), a device by means of which a pair of stereoscopic prints is pinned to a board, and by the simplest of processes viewed stereoscopically with a couple of measuring dots in contact with the picture. These dots merge into a single dot appearing to float at a given distance. Their separation may be varied with a micrometer screw, and this micrometer is set according to the parallax of the pictures, for any desired elevation. Work done with the stereocomparagraph can be quite accurate, providing the pictures have been first restituted accurately; that is that the pictures have been corrected for the tilt of the camera at the instant of exposure. Even without restitution, a reasonably accurate job can be done by parallax methods, providing a great abundance of control has been established. However, it is generally accepted that the stereocomparagraph is inefficient for precision work.

The Brock Process.—The parallax principle is also incorporated into a very precise system known as the Brock process. This system involves the substitution of glass plates which have been carefully restituted for the simple paper prints of the stereocomparagraph method. The stereoscope by which the image is viewed is an extremely precise affair. With the multiplex and the stereoplanigraph methods, the entire map is drawn from beginning to end in one operation and in one machine, whereas the Brock system breaks this process down into a number of operations. This has the advantage of simplicity in that less highly skilled personnel is required, and it also affords opportunity for more operators to work simultaneously on different steps of the operation for each plate pair. The first major step of the Brock process is to determine the tilt of the aerial photograph, and to make a restituted aerial photograph, on glass. Next, this pair of stereoscopic glass plates is placed in the stereoscopes, and the same floating mark is made to follow the contours by an operator turning two wheels. The operator actually causes the floating mark to traverse perhaps one-half inch of the picture, and then with pen and ink draws this half inch of contour on a piece of overlaid tracing paper or acetate sheeting. Then he proceeds to cause the floating mark to indicate another half inch of contour, and draws this with pen and ink. After all the contours from one plate pair have thus been drawn, the result is put in a projecting machine, and each contour is projected at the proper ratio to transform it from the conic to the orthographic projection. The contour is thus traced off, and at the same time the planimetric detail, such as roads and streams, are plotted off, the detail falling between a given pair of contours being drawn at the setting of each contour. Thus, if a 20-ft contour is being traced, the planimetric detail falling within half a contour interval each side of the contour would be drawn at this same setting.

Three-Dimensional Processes

The stereocomparagraph and the Brock processes are two-dimensional methods which measure the parallax of the conic projection all in the horizontal plane and subsequently convert to the orthographic projection by differential enlargement.

Multiplex System.—Perhaps the most widely known of the three-dimensional processes is the multiplex system, which has been adopted by the U. S. Geological Survey, and by the U. S. Corps of Engineers. The photographs are reduced to a small glass diapositive about 2.25 in. square. This miniature, which is one fourth the original size, is then placed in a small projector. Its stereoscopic counterpart is placed in a second adjacent projector, and the two images are projected simultaneously on to a screen of adjustable height. On this screen the two images merge into a three-dimensional miniature of the ground. The two projections are made through complimentary colored filters, and the operator wears spectacles with a red and a blue glass, whereby each eye sees its separate image. Thus a stereoscopic effect is obtained. On the screen is a tiny illuminated dot, called the "floating mark," which rises and falls with the screen. The screen is carried by a stand, which can be pushed around over the table on which the map paper is placed. In the center of this stand is a pencil. The screen is elevated by a thumbscrew to a height representing a desired contour at the scale of the map (1:10,000, for example) and then the stand is pushed around under the direction of the observer, who causes it to move in such a fashion that the floating dot follows the undulating surface of the ground. This floating dot, moving parallel to the horizontal surface of the table, can only move in this one contour level. Thus the operator, by moving the dot continuously over the miniature surface of the ground, getting neither below it nor above it, causes the pencil to draw that particular contour. When this contour is completely drawn, the thumbscrew is turned and the screen and floating dot are raised to the next contour elevation. For example, if the contours are 20 ft apart, the screen and floating dot will be raised by the amount 20 ft ÷ 10,000 or 0.002 ft, which is 0.024 in. Planimetric details, such as roads and streams, are drawn by causing the floating dot to follow the stereoscopic image of these features, rising and falling with the feature by manipulating the thumbscrew.

There are several hundred multiplex units in the various mapping agencies of the United States government, and this system, by reason of its general familiarity, is unquestionably the most popular in the United States.

Stereoplanigraph.—Another three-dimensional method for stereoscopic contour mapping is the stereoplanigraph method,[2] which involves the same general type of stereoscopic pictures as all the other methods. The machine is by far the most elaborate of any of those described, and the most difficult to operate. Its compensating features are great speed and accuracy.

The stereoplanigraph uses a pair of glass diapositives the same size as the original photographs. As in the case of the multiplex, these two diapositives are placed in projectors, although the projectors are four times as large, and consequently four times as far apart. The image from these photographs is projected down on to mirrors, on which is the ubiquitous floating mark. Image and floating mark are observed through an enlarging optical system, by means of which the image appears from five to ten times its original size.

[2] "Stereo-Topographic Mapping," by C. H. Birdseye, *Transactions*, Am. Soc. C. E., Vol. 98 (1933), p. 771.

Contours are traced by an operator turning a pair of hand wheels—the left-hand wheel controlling the "X" motion, and the right-hand wheel controlling the "Y" motion. Elevation is changed by a foot wheel. Drawing is done on a separate table connected with the stereoplanigraph. This table has an X and a Y slide actuating a pencil, under the surveillance of an assistant. Thus the stereoplanigraph is a two-man machine, whereas all the others heretofore described are operated by only one person.

The stereoplanigraph method is similar, in function, to the multiplex, in that the complete operation performed on one plate pair consists of first making the stereoscopic model; that is, bringing the two pictures into perfect correspondence by tipping them and tilting them, and ultimately, by trial and error, returning them to the exact relative position they occupied in the air. In the Brock process this same thing is accomplished by the first step of making the diapositive, the position of which has been determined by calculation. Then, in the multiplex and stereoplanigraph operations, the pair is "horizontalized." This means that the two pictures which have been placed into perfect relative position are now set in their correct relationship to the horizontal reference plane. Then the next step in both the multiplex and stereoplanigraph operations is to draw all the detail on the map. The stereoplanigraph differs most widely from the multiplex at this stage of the proceedings. Whereas the multiplex operator is pushing his screen stand, or pencil stand, around over the map, causing it to draw, the stereoplanigraph operator is doing his drawing by turning a pair of hand wheels. It is the speed of this hand-wheel operation, the fine accuracy, and the optics of the stereoplanigraph, which are responsible for the much greater production per man-hour than can be achieved with any other type of machine. Production, per man-hour, with the stereoplanigraph, on comparative types of mapping, is from five to ten times faster than on the multiplex.

Comparison of Operating Costs

A comparison of the merits of the various systems described briefly herein involves, first, an understanding of some of the fundamental criteria used, in evaluating the work.

The most efficient system of mapping is that by which a given map made to a given degree of accuracy can be produced cheapest and fastest. The principal comparative items of cost in compiling a contour map by photogrammetric processes may be classified roughly as follows: (1) Amortization of the capital investment; (2) flying cost; (3) control cost; and (4) plotting cost.

Other items of cost, such as drafting and the routine editing, are more or less similar for all systems of mapping, and consequently do not constitute one of the bases of comparison.

(*1*) *Amortization of Cost.*—The stereocomparagraph costs about $300. The Brock equipment is not listed, only one unit of this equipment being in existence. The writer estimates that it might cost $25,000 or $30,000 to duplicate one of the Brock units. The multiplex costs about $10,000 a unit; and the stereoplanigraph costs about $45,000 per unit. Assuming that the equipment will be used for ten years, there is no amortization to be worried about on the

stereocomparagraph; the multiplex must be written off at the rate of $1,000 per yr; the Brock equipment at the rate of about $2,500 per yr; and the stereoplanigraph at the rate of about $4,500 per yr. On the assumption that the equipment will be used only one shift (say, 2,000 hr per yr) the multiplex has an amortization charge of 50¢ per hr; the Brock equipment has an amortization charge of $1.25 per hr; and the stereoplanigraph an amortization charge of $2.25 per hr.

(*2*) *Flying.*—Since the stereocomparagraph is rarely, if ever, used for precision mapping, the remaining comparison will be confined to the other processes and instruments. For multiplex work, flying is generally undertaken at an elevation of from 700 to 1,100 times the permissible tolerance in the contours, as compared with 2,500 times for the stereoplanigraph. The net area covered per photograph can be compared on the following basis: The multiplex method covers the least area per photograph. Thus, if the multiplex covers 810 units of area per stereoscopic model to a given degree of accuracy, the stereoplanigraph method will cover 1,620 units of area. These figures can be clarified somewhat by stating that the multiplex, flying 1,000 times the permissible tolerance, uses a 6-in. lens covering a 9-in. by 9-in. photograph. The stereoplanigraph flies at 2,500 times the permissible tolerance, using a 6-in. lens on a 9-in. by 9-in. negative. The different systems also use a different overlap which is taken into consideration in the foregoing figures. The number of pictures required and the degree of control required will be in the following ratio: Stereoplanigraph 0.64 and multiplex 2.00.

The relative separation between the photographic strips will be 7.2 for the multiplex, as compared with 14.4 for the stereoplanigraph. The actual amount of flying to be done will be inversely proportional to these figures; in other words, the most flying must be done for the multiplex work.

(*3*) *Control.*—Although there is some difference of opinion among experts as to the most economical method to control a photogrammetric survey, the cost of doing the work for the different methods, according to the standard of any expert, will be substantially in the same proportion as the area covered per photograph. Control generally involves about four known points per stereoscopic pair. Since most of these points are common to a number of stereoscopic "models," the net by almost all systems will prove to be approximately one and one-half control points per picture. So, by almost any conceivable method of control, the relative cost will be approximately twice as much per unit of area for the multiplex method as for the stereoplanigraph method. (Note: It should be kept in mind that the writer is most familiar with the stereoplanigraph process. Much has been published on the multiplex process, and persons familiar with the multiplex system will have an opportunity to discuss this paper. Little has been published as to the cost of the Brock process, and it is hoped that someone thoroughly familiar with the Brock process will take this opportunity to present its case in the form of a discussion.)

(*4*) *Plotting the Map.*—The fact that a map can be plotted by stereoplanigraph from five to ten times as fast as by multiplex is due to many factors. First and foremost is the relative speed of the hand-wheel control as compared

with the clumsy job of pushing the multiplex stand around by hand. Second, the time is affected by the greater area per stereoscopic model which is available to the stereoplanigraph, and consequently less time is needed in the preparatory setting up work, as it takes about as long to set up one model as another, regardless of scale or flying altitude. Another advantage of the stereoplanigraph is that it affords a means of enlargement between the picture scale and the plotting scale. Usually the map is drawn directly by the stereoplanigraph at a scale about six times larger than the original picture scale. The multiplex, on the other hand, is at a distinct disadvantage in that it first reduces the pictures to one-fourth size, and then has a limited range of enlargement due to the shallow depth of focus of the multiplex projectors. Therefore, it will be seen that, in plotting the same map, the multiplex operator actually will have to draw from two to three times the number of linear inches of contour that the stereoplanigraph operator will have to draw.

Although no data are available on the actual rate of drawing in the Brock stereoscope, the writer believes that the drawing is probably the slowest of any of the three methods. The Brock process does not permit the continuous drawing of a contour line, as do the other two processes. In the Brock process, the contour line must be drawn by short increments, the operator turning the hand wheels of the stereoscope and observing where the contour line goes for a short distance such as one-half inch, and then drawing that contour line by hand. Then he observes another half inch and draws that by hand.

There is another factor which enters the cost of the drawing by the three machines, however: Whereas the stereoplanigraph permits much faster drawing, it also can be operated only by a much higher type of personnel, the training period for efficient operators being about two years. One of the great advantages of the multiplex is that the method is simple and very apparent, and that reasonably alert operators can be trained in probably one quarter of the time that it would take to train them for the stereoplanigraph. In this respect the Brock process has an advantage over both other methods in that an ordinary topographical draftsman with good eyes can be taught to become a Brock operator in a comparatively short period of time—probably in one tenth the time that it would take to train a stereoplanigraph operator. The reason for this is that the Brock plates come to the operator with the tilt entirely eliminated, so that all the operator has to do is to set them in the machine and start to draw. On the other hand, both the stereoplanigraph and the multiplex operators must learn to set up the stereoscopic model and to horizontalize it—processes which take a considerable period of time to master. The stereoplanigraph must be set up with the greatest precision of any machine, because the pictures are taken at such very high altitudes and are observed under such high magnification. The optical qualities of the multiplex image are not nearly so good as those of the stereoplanigraph, and consequently the stereoscopic model cannot be established with the same degree of precision. Therefore the instrument can be set up more quickly and with less experience.

In summary, all three methods will produce an excellent map. The multiplex appears to be the least efficient from the standpoint of the flying and

control required, and second in the time required to plot. The stereoplanigraph will actually permit plotting the map much faster per man-hour than the Brock process; but in so doing it utilizes a machine that has a high rate of amortization. It involves an operator and an assistant to run the machine, in comparison to a single lower-salaried operator on the Brock process. If the stereoplanigraph can be kept in use continually, so that the amortization charge is not a major factor, it is the most efficient method now known, by a considerable margin. However, if the machine is only to be used occasionally, it is conceivable that the Brock process, or even the multiplex, if the period of use compared with the total available period is small enough, might be more efficient, due to the amortization charges.

Comparison with Ground Methods

So far, this paper has considered only a comparison of the different photogrammetric methods. The subject could not be considered thoroughly covered without making some comparison between photogrammetric methods and the older and more conventional ground method—that is, the plane-table method—of drawing a contour map. There are undoubtedly many situations in which the plane table is more efficient than any photogrammetric method. Any very small area can be surveyed more economically by a plane table. The job is done completely and finished by the man in the field. With the photogrammetric method, it is necessary first to send an airplane to the area to be photographed. Subsequently control parties are sent out to establish ground control, and finally the data are put through a machine to produce the map. Just where this line of demarcation is between the efficiencies of the two, according to scale and number of acres, is difficult to determine. The smaller and farther away an area is from the flying base, the more relative efficiency favors the plane-table method.

As an hypothetical illustration, consider an area of four hundred acres right beside the flying field, to be done either by plane-table methods or by photogrammetric methods for a map scale of 1 in. = 200 ft, and 5-ft contours. If this area were rolling to mountainous, the photogrammetric method probably would only cost half as much as the plane-table method. On the other hand, if it were reasonably flat to rolling, and open, the costs would probably be about the same. Remove this same four hundred acres two hundred miles away from the flying field, and the costs by the two methods might be about the same. The cost of using the airplane will be perhaps $50 an hr, and five hours may be required to complete the job—providing the photographers record all the data correctly on the first attempt. Perhaps, also, an allowance of 50% should be made for possible re-takes, which would be 7½ hr, at $50 per hr. Thus, there is a charge of about $1 an acre for the flying, really before the work is started.

It may also be said that, in areas which are very flat with contours far apart, plane-table methods probably are more efficient than any photogrammetric method.

The plane-table method may be more efficient in the case of a road loca-

tion which had already been substantially staked out; for example, for a strip of topography only 100 ft wide and 20 miles long. In this instance, the photogrammetric survey would cost almost as much as if the width of the strip to be mapped were the entire width that could be covered by a picture, which might be 2,000 ft or 3,000 ft. The plane-table operator can confine his cost to the actual width of 100 ft. Consequently, there are many cases where, in locational surveys, it will be found more economical to run a plane-table survey, or to cross section the location, than to make a contour map by photogrammetric methods. This is not universally true, however, as many circumstances, such as accessibility to the ground, etc., will often favor the photogrammetric method.

It can be stated definitely that for final determination of canal grades, where the fall of the canal must be confined to a few inches per mile, photogrammetric methods are of no avail except in the preliminary stages of the survey. The final location must be established by level.

Summary

Today in the United States it is probable that nearly all topographic mapping is being done by photogrammetric methods. The U. S. Geological Survey is the only organization among many now engaged upon the federal mapping program that still utilizes the plane-table method to any extent. This organization operates what is probably the largest multiplex unit in the world, so that even within the Geological Survey a large percentage of the total work is being done by photogrammetric methods. Other federal organizations and private organizations under contract to the government use the photogrammetric methods almost exclusively.

During the cantonment construction program for World War II, when detailed contour maps were required in a great hurry, the photogrammetric method was not found wanting. The speed of photogrammetric mapping has proved a surprise to people unfamiliar with the methods, and the uniform accuracy of the results has made design from photogrammetric maps a pleasure to the engineer. Photogrammetric maps are being made with a guarantee that 90% of all points checked shall be within one-half contour interval of their true position. Photogrammetric maps have been made at scales ranging from 1 in. = 20 ft, with 1-ft and 2-ft contours, to scales of 1:62,500, with 100-ft contours.

Photogrammetry has finally arrived at the age of full maturity. Although the methods will unquestionably continue to improve from year to year, they are already thoroughly practicable, and afford a tool which will find continuing and increased acceptance at the hands of thoughtful engineers.

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PAPERS

DETERMINATION OF PRESSURES WITHIN A SIPHON SPILLWAY

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Synopsis

A correct determination of the internal pressures in closed conduits is an important factor in their design. In ordinary pipes with a straight alinement the computation of internal pressures is rather simple. It is limited to determination of the height of the piezometric line above the center line of the pipe after which a multiplication of this piezometer height by the unit weight of water gives the average value of the internal pressure within the pipe. This pressure is normally called "positive"—that is, in regard to the pressures exerted by air it is greater than the latter and their difference is positive. However, there are cases in which pressures become "negative," or smaller than the pressure exerted by air. Such cases occur (*a*) in closed conduits when the vertical alinement is above the piezometric line, (*b*) in conduits behind valves or gates; and (*c*) in siphons in which the effect of the curvature of the flow filaments is revealed by a distribution of internal pressures different from hydrostatic pressures. In the siphons the pressures may become either positive or negative. Expressions for the pressures within a siphon are derived in this paper.

Introduction

In designing a true siphon the strength of the walls should be based on pressures within the siphon, because they may become smaller or greater than atmospheric pressure. The unbalanced pressures resulting from the curvilinear flow will produce forces which will be especially noticeable at the summit and at the outlet, provided the latter has the usual reverse curves.

Notation.—The letter symbols used in this paper are defined where they first appear in the paper, and are assembled for reference in the Appendix.

Note.—Written comments are invited for immediate publication; to insure publication the last discussion should be submitted by **August 1, 1945.**

[1] With U. S. Engr. Dept., Sacramento, Calif.

THEORY

The theory upon which the derivations were based is that of the free vortex or irrotational flow with a velocity potential.[2] With this assumption the energy content E for each filament is the same, independent of the location of the filament within the flow. By inspection in Fig. 1, the atmospheric

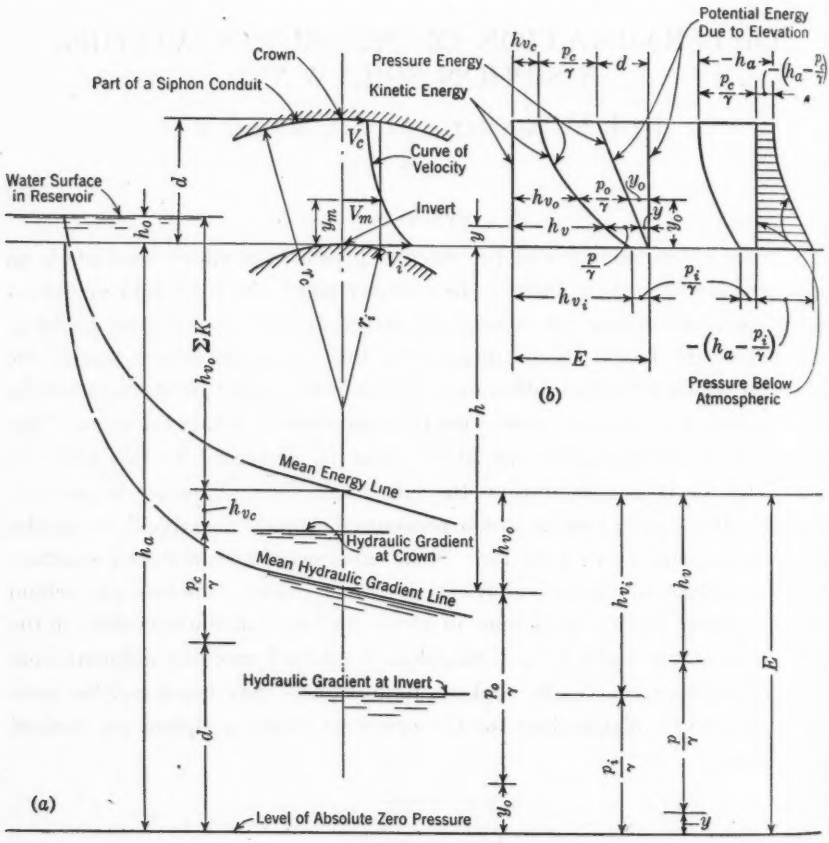


FIG. 1.

vacuum head h_a is equal to

$$h_a = -h + y + \frac{p}{\gamma} + h_v - h_{vo} \quad \ldots\ldots\ldots\ldots\ldots\ldots (1a)$$

or

$$\frac{p}{\gamma} = h_a + h - y - h_v + h_{vo} \quad \ldots\ldots\ldots\ldots\ldots\ldots (1b)$$

in which: h = the distance from the invert to the mean water gradient (positive

[2] Discussion by B. A. Bakhmeteff of "Tests of Broad-Crested Weirs," by J. G. Woodburn, *Transactions*, Am. Soc. C. E., Vol. 96 (1932), p. 430.

when the mean water gradient is above the invert); y = the height above the invert to a filament where a value of pressure is to be determined; p = pressure intensity at the filament, distant y above the invert; h_v = velocity head at the filament distant y above the invert; h_{vo} = mean velocity head corresponding to pressure energy $\frac{p_o}{\gamma}$ and distance above the invert y_o; and γ = the unit weight of water = 62.5 lb per cu ft.

Likewise, by inspection in Fig. 1,

$$\frac{p}{\gamma} = h_{vi} + \frac{p_i}{\gamma} - h_v - y \quad \ldots\ldots (2)$$

in which h_{vi} = velocity head at the filament along the invert. Combining Eqs. 1*b* and 2:

$$\frac{p_i}{\gamma} = h_a + h - h_{vi} + h_{vo} \quad \ldots\ldots (3)$$

Once $\frac{p_i}{\gamma}$ is found, $\frac{p_c}{\gamma}$, $\frac{p_o}{\gamma}$, $\frac{p_i}{\gamma}$ and $\frac{p}{\gamma}$ can be determined from the following expression (see Fig. 1):

$$h_{vc} + \frac{p_c}{\gamma} + d = h_{vo} + \frac{p_o}{\gamma} + y_o = h_v + \frac{p}{\gamma} + y = h_{vi} + \frac{p_i}{\gamma} = E \quad \ldots\ldots (4)$$

from which

$$\frac{p_c}{\gamma} = E - h_{vc} - d \quad \ldots\ldots (5a)$$

$$\frac{p_o}{\gamma} = E - h_{vo} - y_o \quad \ldots\ldots (5b)$$

$$\frac{p_i}{\gamma} = E - h_{vi} \quad \ldots\ldots (5c)$$

and

$$\frac{p}{\gamma} = E - h_v - y \quad \ldots\ldots (5d)$$

In Eqs. 3, 4, and 5, p_c (at the crown), p_o (at y_o), and p (at any point y) are absolute pressures; that is, the atmospheric pressure has been added in each case. The pressures relative to the atmospheric pressure p_r can be obtained by deducting h_a from the absolute pressures p_a as shown in Fig. 1(*b*), or:

$$p_r = p_a - h_a \gamma \quad \ldots\ldots (6)$$

These relative pressures (p_r) are the ones to be used in structural computations. (In this paper, a minus sign indicates that the resulting pressure is directed toward the inside of the conduit, and a plus sign means the reverse.)

For a complete solution it is necessary to determine h_a, h, h_{vi}, h_{vc} (velocity head at the crown), h_{vo}, and the dimensions of the siphon cross sections—such as the width b, the depth or height d of a cross section normal to the direction of flow, and one of the radii. Let h_a = 24 ft to 28 ft as reduced from its theoretical value of 33.9 ft due to vapor pressure. Other values of head —h, h_{vi}, h_{vc}, and h_{vo}—are determined from flow computations.

Assume that the mean velocity V_m (equal to the velocity at distance y_m

above the invert) and the cross section area A are given. First, the flow will be referred to mean hydraulic gradient and mean energy gradient, thus:

$$h = h_o - (1 + \sum K)\, h_{vo} \quad \ldots\ldots (7)$$

in which h_o = the head necessary for priming the siphon or in general the head above the invert of the summit of a siphon and K = the coefficient of loss from the inlet to the point under consideration. Usually the value of h or the position of the mean hydraulic gradient is fixed by the conditions at the outlet, whether free or submerged, and with the discharge defined by these conditions the mean velocity V_m can be found. Then the mean velocity head entering Eq. 7 must be expressed in terms of the mean velocity.

A coefficient $n = \dfrac{h_{vo}}{h_{vm}}$, representing the ratio of mean velocity head to that due to mean velocity, can be determined from the expression:

$$h_{vo} = n\, h_{vm} = \frac{\int_0^d \frac{V^2}{2g}\, dq}{\int_0^d V\, dy} = \frac{\int_0^d \frac{V^2}{2g}\, V\, dy}{\int_0^d V\, dy} = \frac{\int_0^d \frac{V^3}{2g}\, dy}{\int_0^d V\, dy} \quad \ldots\ldots (8)$$

in which h_{vm} = the velocity head due to a mean velocity V_m; V = the velocity at any filament distant y from the invert; g = the gravity constant; and q = the discharge per unit width. For a flow convex upward, the velocity at any filament y is:

$$V = V_c \frac{r_i + d}{r_i + y} = V_c \frac{r_c}{r_i + y} \quad \ldots\ldots (9)$$

in which V_c = the velocity at the filament at the crown; r_i = the radius of the siphon at the invert (Fig. 1); and r_c = the radius of the siphon at the crown. Substituting Eq. 9 in Eq. 8 and writing the solution for n:

$$n = \frac{h_{vo}}{h_{vm}} \quad \ldots\ldots (10a)$$

that is:

$$n = \frac{\int_0^d \frac{V_c^3}{2g} \left(\frac{r_c}{r_i + y}\right)^3 dy}{\frac{V_m^2}{2g} \int_0^d V_c \left(\frac{r_c}{r_i + y}\right) dy} = \left(\frac{V_c}{V_m}\right)^2 \frac{\int_0^d \left(\frac{r_c}{r_i + y}\right)^3 dy}{\int_0^d \left(\frac{r_c}{r_i + y}\right) dy} \quad \ldots\ldots (10b)$$

Let $\dfrac{V_c}{V_m} = \beta$ and $\dfrac{r_c}{r_i} = m > 1$; then Eq. 10*b* becomes

$$n = \beta^2 r_c^2 \frac{\int_0^d \frac{dy}{(r_i + y)^3}}{\int_0^d \frac{dy}{r_i + y}} = \beta^2 r_c^2 \frac{-0.5\,(r_i + y)^{-2} \Big|_0^d}{\log_e (r_i + y) \Big|_0^d}$$

$$= -\frac{\beta^2 r_c^2}{2} \frac{r^{-2}_c - r^{-2}_i}{\log_e r_c - \log_e r_i} = \frac{\beta^2 (m^2 - 1)}{2 \log_e m} \quad \ldots\ldots (11)$$

For a flow concave upward, the velocity at any filament distant y above the invert is:

$$V = V_c \frac{r_i - d}{r_i - y} = V_c \frac{r_c}{r_i - y} \quad \ldots\ldots\ldots\ldots\ldots\ldots (12)$$

Substituting Eq. 12 in Eq. 8 and again writing the solution for n:

$$n = \frac{h_{vo}}{h_{vm}} = \frac{\int_0^d \frac{V_c^3}{2g}\left(\frac{r_c}{r_i - y}\right)^3 dy}{\frac{V_m^2}{2g}\int_0^d V_c \frac{r_c}{r_i - y}\, dy} = \left(\frac{V_c}{V_m}\right)^2 \frac{\int_0^d \left(\frac{r_c}{r_i - y}\right)^3 dy}{\int_0^d \frac{r_c}{r_i - y}\, dy}$$

$$= \beta^2 r_c^2 \frac{\int_0^d \frac{dy}{(r_i - y)^3}}{\int_0^d \frac{dy}{r_i - y}} \quad \ldots\ldots\ldots\ldots\ldots\ldots (13)$$

As with Eq. 10, when Eq. 13 is written in terms of β and m, the algebraic result is identical with Eq. 11. In applying this formula, however, a different numerical result is obtained for the case of flow convex upward (in which m is greater than unity) than for the case of flow concave upward (in which m is less than unity). Now to determine the expression for β: Since

$$V_m = \frac{q}{d} = \frac{\int V\, dy}{d} \quad \ldots\ldots\ldots\ldots\ldots\ldots (14)$$

the value of $\beta = \frac{V_c}{V_m}$, for the case of flow convex upward, is determined from the following expression:

$$\beta = \frac{V_c}{V_m} = \frac{V_c\, d}{\int_0^d V\, dy} = \frac{V_c\, d}{\int_0^d V_c \frac{r_i + d}{r_i + y}\, dy}$$

$$= \frac{d}{r_c} \frac{1}{\log_e r_c - \log_e r_i} = \frac{d}{r_c \log_e \frac{r_c}{r_i}} = \frac{1}{\frac{r_c}{d} \log_e m} \quad \ldots\ldots\ldots\ldots (15)$$

in which $m > 1$. For the case of flow concave upward it is:

$$\beta = \frac{V_c\, d}{\int_0^d V_c \frac{r_i - d}{r_i - y}\, dy} = -\frac{d}{(r_i - d) \log_e (r_i - y)\Big|_0^d}$$

$$= \frac{-d}{r_c \log_e \frac{r_i}{r_c}} = \frac{1}{\frac{r_c}{d} \log_e \frac{1}{m}} \quad \ldots\ldots\ldots\ldots\ldots\ldots (16)$$

in which $m < 1$. Eqs. 15 and 16 are not identical, and they do not render identical numerical results.

The relationships between the various velocity heads are as follows. By definition (see Eq. 10*a*) $h_{vo} = n\,h_{vm}$. The corresponding relationship between h_{vc} and h_{vm} is defined as:

$$h_{vm} = \frac{V^2_m}{2g} \times \frac{V^2_c}{V^2_c} = \frac{V^2_c}{2g}\left(\frac{V_m}{V_c}\right)^2 = \frac{1}{\beta^2}\,h_{vc} \quad \ldots\ldots\ldots\ldots (17a)$$

Likewise,

$$h_{vc} = \frac{V^2_c}{2g} \times \frac{V^2_m}{V^2_m} = \frac{V^2_m}{2g}\left(\frac{V_c}{V_m}\right)^2 = \beta^2\,h_{vm} \quad \ldots\ldots\ldots\ldots (17b)$$

The relationship between h_{vi} and h_{vm} is defined as follows, since $\frac{V_i}{V_c} = \frac{r_c}{r_i} = m$,

$$h_{vi} = \frac{V^2_i}{2g} \times \frac{V^2_c}{V^2_c} = \frac{V^2_c}{2g}\left(\frac{V_i}{V_c}\right)^2 = h_{vc}\,m^2 = \beta^2\,m^2\,h_{vm} \quad \ldots\ldots\ldots (18)$$

The relationship between h_v, h_{vc}, h_{vm}, and h_{vi} for the flow convex upward is as follows: $\frac{V}{V_c} = \frac{r_c}{r_i + y}$ or

$$h_v = h_{vc}\left(\frac{r_c}{r_i + y}\right)^2 = \beta^2\,h_{vm}\left(\frac{r_c}{r_i + y}\right)^2 = \frac{h_{vi}}{\left(1 + \frac{y}{r_i}\right)^2} \quad \ldots\ldots (19a)$$

In the same way, for the case of flow concave upward:

$$h_v = h_{vc}\left(\frac{r_c}{r_i - y}\right)^2 = \beta^2\,h_{vm}\left(\frac{r_c}{r_i - y}\right)^2 = \frac{h_{vi}}{\left(1 - \frac{y}{r_i}\right)^2} \quad \ldots\ldots (19b)$$

The position of the filament at which the mean velocity head h_{vo} occurs is determined as follows: For the flow convex upward, since $h_{vo} = n\,h_{vm}$ (see Eq. 10*a*),

$$h_{vo} = n\,h_{vm} = h_{vc}\left(\frac{r_i + d}{r_i + y_o}\right)^2 \quad \ldots\ldots\ldots\ldots\ldots (20a)$$

or, since $h_{vc} = \beta^2\,h_{vm}$,

$$n = \frac{\beta^2\,r^2_c}{(r_i + y_o)^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (20b)$$

Therefore,

$$y_o = \frac{\beta\,r_c}{\sqrt{n}} - r_i = r_i\left(m\sqrt{\frac{2\log_e m}{m^2 - 1}} - 1\right) \quad \ldots\ldots\ldots\ldots (21)$$

For the flow concave upward:

$$h_{vo} = n\,h_{vm} = h_{vi}\,\frac{r^2_c}{(r_i - y_o)^2} \quad \ldots\ldots\ldots\ldots\ldots (22a)$$

or

$$n = \beta^2\left(\frac{r_i - d}{r_i - y_o}\right)^2 = \frac{\beta^2\,r^2_c}{(r_i - y_o)^2} \quad \ldots\ldots\ldots\ldots\ldots (22b)$$

and

$$y_o = -\left(\frac{r_c}{\sqrt{n}} - r_i\right) = r_i\left(1 - m\sqrt{\frac{2\log_e m}{m^2 - 1}}\right) \quad \ldots\ldots\ldots (23)$$

Example 1. Pressures at the Crown and Invert of the Summit Section (Convex Upward).—Consider a siphon for which the general conditions are as listed in Table 1(*a*). As shown in Table 1(*b*), other basic quantities are: $r_c = 20$ ft

TABLE 1.—COMPUTATIONS FOR PRESSURES AT THE CROWN AND INVERT OF A SIPHON

(*a*) GENERAL DATA

$Q = 100$ cu ft per sec, $A = 10$ sq ft, $b = 2.5$ ft, $d = 4$ ft, $r_i = 16$ ft, $q = 100/2.5 = 40$ cu ft per sec, $V_m = 40/4 = 10$ ft per sec, $h_{vm} = 1.55$ ft, and $h_a = 24$ ft

Equation	Symbol	Computation	Quantity	Unit
		(*b*) EXAMPLE 1.—SUMMIT SECTION (CONVEX UPWARD, $h = -10$ FT)		
...	r_c	$16 + 4$	20	Ft
...	m	$r_c/r_i = 20/16$	1.25	...
15	β	$1/(5 \times 0.223)$	0.90	...
17	h_{vc}	$0.90^2 \times 1.55$	1.26	Ft
18	h_{vi}	$1.25^2 \times 1.26$	1.96	Ft
11	n	$\frac{0.90^2 (1.25^2 - 1)}{2 (\log_e 1.25)}$	1.025	...
10*a*	h_{vo}	1.025×1.55	1.59	Ft
3	p_i/γ	$24 - 10 - 1.96 + 1.59$	13.63	Ft
...	p_i	$13.63 \times 62.5 \times 1/144$	5.93	Lb per sq in.
5*c*	E	$1.96 + 13.63$	15.59	Ft
5*a*	p_c/γ	$15.59 - 1.26 - 4$	10.33	Ft
...	p_c	$10.33 \times 62.5 \times 1/144$	4.48	Lb per sq in.
21	y_o	$16\left(1.25\sqrt{\frac{2 \log_e 1.25}{1.25^2 - 1}} - 1\right)$	1.81	Ft
5*b*	p_o/γ	$15.59 - 1.59 - 1.81$	12.19	Ft
...	p_o	$12.19 \times 62.5 \times 1/144$	5.33	Lb per sq in.
...	p_a	$24 \times 62.5 \times 1/144$	10.42	Lb per sq in.
		(*c*) EXAMPLE 2.—SECTION NEAR THE OUTLET (CONCAVE UPWARD, $h = +10$ FT)		
...	r_c	$16 - 4$	12	Ft
...	m	$r_c/r_i = 12/16$	0.75	...
16	β	$1/(3 \times 0.288)$	1.15	...
17	h_{vc}	$1.15^2 \times 1.55$	2.06	Ft
18	h_{vi}	$0.75^2 \times 2.06$	1.16	Ft
11	n	$\frac{1.15^2 (0.75^2 - 1)}{2 (\log_e 0.75)}$	1.01	...
10*a*	h_{vo}	1.01×1.55	1.58	Ft
3	p_i/γ	$24 + 10 - 1.16 + 1.58$	34.42	Ft
...	p_i	$34.42 \times 62.5 \times 1/144$	14.90	Lb per sq in.
5*c*	E	$1.16 + 34.42$	35.58	Ft
5*a*	p_c/γ	$35.58 - 2.06 - 4$	29.52	Ft
...	p_c	$29.52 \times 62.5 \times 1/144$	12.78	Lb per sq in.
23	y_o	$16\left(1 - 0.75\sqrt{\frac{2 \log_e 0.75}{0.75^2 - 1}}\right)$	2.19	Ft
5*b*	p_o/γ	$35.58 - 1.58 - 2.19$	31.81	Ft
...	p_o	$31.81 \times 62.5 \times 1/144$	13.78	Lb per sq in.
...	p_a	$24 \times 62.5 \times 1/144$	10.42	Lb per sq in.

and $m = 1.25$. The remaining steps of the solution follow in the sequence of steps in Table 1(*b*).

By Eq. 6 and Table 1(*b*) the pressures relative to atmospheric pressure, in pounds per square inch, are:

$$p_{cr} = 4.48 - 10.42 = -5.94$$
$$p_{or} = 5.33 - 10.42 = -5.09$$
$$p_{ir} = 5.93 - 10.42 = -4.49$$

The direction of action of these pressures is inward—that is, toward the inside of the conduit. The diagram of distribution of the absolute and relative pressures can be obtained from Eqs. 5*d* and 19*a*, thus:

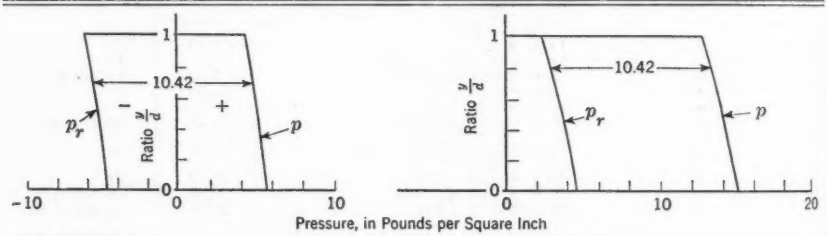
$$p = (E - h_v - y)\,\gamma = \frac{\gamma}{144}\left[E - \frac{h_{vi}}{\left(1 + \frac{y}{r_i}\right)^2} - y \right] \quad \ldots\ldots\ldots (24)$$

or, substituting appropriate values,

$$p = \frac{62.5}{144}\left[15.59 - \frac{1.96}{\left(1 + \frac{y}{16}\right)^2} - y \right] = 1.74\left[3.90 - \frac{0.49}{\left(1 + \frac{y}{4d}\right)^2} - \frac{y}{d} \right] \quad (25)$$

in which $\frac{y}{d} = \frac{y}{4}$. The values of p are given in Table 2(*a*).

TABLE 2.—PRESSURES IN A SIPHON, IN POUNDS PER SQUARE INCH; ILLUSTRATIVE EXAMPLES



(*a*) EXAMPLE 1 (*b*) EXAMPLE 2

Item	Pressure	Ratios y/d					
		1.00	0.80	0.60	0.40	0.20	0.00
(*a*) Example 1.—Flow Through the Summit; Convex Upward							
1	p	4.48	4.80	5.10	5.40	5.66	5.93
2	p_r	−5.94	−5.62	−5.32	−5.02	−4.76	−4.49
(*b*) Example 2.—Flow Through the Section Near the Outlet; Concave Upward							
3	p	12.78	13.30	13.75	14.17	14.60	14.90
4	p_r	2.36	2.88	3.33	3.75	4.18	4.48

Example 2. Pressures at the Crown and Invert of a Siphon Section Near the Outlet.—The basic quantities, Q, A, b, k, r_i, q, V_m, h_m, and h_a, are the same as in Example 1. Other basic quantities (see Table 1(*c*)) are: $r_c = 12$ ft and $m = 0.75$. The remaining steps of the solution are listed in Table 1(*c*). By Eq. 6 the pressures relative to atmospheric pressure, in pounds per square inch, are:

$$p_{cr} = 12.78 - 10.42 = 2.36$$
$$p_{or} = 13.78 - 10.42 = 3.36$$
$$p_{ir} = 14.90 - 10.42 = 4.48$$

The direction of these pressures is outward—that is, toward the outside of the conduit. Similar to Example 1, the diagram of absolute and relative pressures is obtained from Eqs. 5*d* and 19*b*; thus (compare Eq. 24):

$$p = (E - h_v - y) = \frac{\gamma}{144}\left[E - \frac{h_{vi}}{\left(1 - \frac{y}{r_i}\right)^2} - y \right] \quad \ldots\ldots\ldots (26)$$

Again, substituting appropriate values,

$$p = \frac{62.5}{144}\left[35.58 - \frac{1.16}{\left(1 - \frac{y}{16}\right)^2} - y \right] = 1.74\left[8.90 - \frac{0.29}{\left(1 - \frac{y}{4d}\right)^2} - \frac{y}{d} \right] \quad (27)$$

in which $\frac{y}{4} = \frac{y}{d}$. The values of p for Example 2 are given in Table 2(*b*).

Summary

The derivations in this paper, which are based on the formulas for an idealized vortex flow in a siphon, and the subsequent numerical examples, illustrate that the pressures both at the summit and at the outlet of a siphon become unbalanced relative to the atmospheric pressures. Therefore, structural design should provide for these unbalanced pressures with due amount of steel reinforcement in the concrete if the siphon is a reinforced concrete structure or with an additional quantity of steel if it is of steel. Actual failures of siphons indicate the necessity for the provisions recommended.

The unbalanced pressures in the examples cited herein are about 5 lb per sq in. maximum, which is about 0.4 ton per sq ft or about 720 lb per sq ft. If it is remembered that a live load created by a crowd of people is usually assumed to be 100 lb per sq ft and in warehouses as much as 400 lb per sq ft, then it will be realized what magnitude of pressure is imposed on siphons and why some of them fail. It is therefore considered that a siphon (or as a matter of fact any closed conduit) with a curvilinear flow should be designed for the unbalanced pressures resulting from such flow.

APPENDIX

Notation

The following letter symbols, used in the paper, conform essentially to American Standard Letter Symbols for Hydraulics (ASA—Z10.2—1942), prepared by a Committee of the American Standards Association, with Society representation, and approved by the Association in 1942.

Without subscripts, the symbols refer to the summit, outlet, or anywhere along the line of the siphon. Subscripts c and i, respectively, denote the crown point and invert at any section.

A = area of siphon cross section;
b = width of the siphon;
c = subscript denoting "crown";
d = height of the cross section under consideration, normal to the direction of flow;
E = energy content as shown in Fig. 1 (see Eq. 4);
g = gravity constant;
h = head = distance from the invert to the mean water gradient (positive when the mean water gradient is above the invert);
h_a = atmospheric vacuum head;
h_o = distance from the invert at the summit of a siphon to the water level in the reservoir;
h_v = velocity head at a filament distant y from the invert:
  h_{vc} = velocity head at the crown;
  h_{vi} = velocity head at the invert;
  h_{vm} = velocity head due to mean velocity;
  h_{vo} = mean velocity head;
i = subscript denoting "invert";
K = coefficient of energy loss from the inlet to the section under consideration;
m = ratio of the radius at the crown r_c to the radius at the invert r_i;
n = ratio of the mean velocity head h_{vo} to the velocity head h_{vm} created by the mean velocity V_m;
p = pressure intensity at a filament distant y above the invert:
  p_a = absolute pressure intensity;
  p_c = pressure at the crown;
  p_i = pressure at the invert;
  p_o = pressure intensity at a filament with mean kinetic energy, distant y_o above the invert;
  p_r = pressure relative to atmospheric pressure;
Q = rate of flow in a siphon;
q = rate of flow per unit width of siphon;
r = radius, with appropriate subscripts;
V = velocity of flow, with appropriate subscripts:
  V_m = velocity at a filament equal to the mean value of V, distant y_m from the invert;
y = distance to any filament measured upward from the invert:
  y_m = distance above the invert where the velocity of a filament is the same as the mean velocity of the section;
  y_o = distance above the inert where the kinetic energy of a filament is the same as the mean kinetic energy of the section;
β = ratio of the velocity of flow at the crown, V_c, to the mean velocity, V_m;
γ = unit weight of water, assumed equal to 62.5 lb per cu ft.

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PAPERS

COMPUTING COAGULANT REQUIREMENTS IN SLUDGE CONDITIONING

BY A. L. GENTER,[1] M. AM. SOC. C. E.

SYNOPSIS

The practice of partly dewatering sewage sludges in continuous vacuum filters is relatively recent. The procedure for estimating the initial cost of such an installation for different types of sludges is comparatively simple. It involves estimating the tons of sludge solids to be disposed of during each operating day and the capacity and dimensions of the component mechanical elements of the filtration equipment. The mathematical basis for the latter factors has been supplied by rather extensive experience with such equipment.

However, when it comes to the more important item of predicting the annual operating charges involved in using this equipment, a logical mathematical basis is lacking. This is due to the fact that experience with such filters has demonstrated the necessity of using chemical reagents to render all typical sludges sufficiently curdled or coagulated to permit their rapid draining under suction. This experience has shown that the sludge demand for such reagents varies greatly with various types of sludges in different plants; and, even in the same plant, filter yields are largely a function of the amount of coagulant added to the sludge, providing the drum speed of the continuous filter is adapted to certain ranges of coagulant dosage.

Empirical knowledge of these items relating to chemical requirements has been gained by methods of trial and error and involves assumptions that need clarification by mathematical analysis. At times these assumptions have been disappointing from the standpoint of annual operating costs. In quite a number of plants the cost of chemicals per ton of sludge solids treated is an extremely expensive item. Instances are also known in which such filters have been installed and then either abandoned or partly substituted by less expensive methods of sludge lagooning, sand-bed dewatering, or trucking of liquid sludge

NOTE.—Written comments are invited for immediate publication; to insure publication the last discussion should be submitted by **August 1, 1945.**

[1] Cons. Engr., Baltimore, Md.

to neighboring farm areas because the annual operating charges for chemicals and incidental filter upkeep were unexpectedly high.

The objective of this paper is to present some evidence of a mathematical relationship between the quantities of chemical reagents demanded by various sludges and certain basic sludge assays. Such a relationship should accomplish several useful purposes:

(*a*) It should supply an explanation of where chemical coagulants go in dosing typical sludges;
(*b*) It should enable a designing engineer to select the most economical method of sludge processing and disposal for given local conditions;
(*c*) It should enable the plant superintendent to determine what may be done to improve the economics of sludge conditioning and filtration in his plant; and
(*d*) It should stimulate further investigation of the factor presented herein relating to the influence of volatile matter on coagulant demand.

As ferric chloride remains the most effective coagulant—that is, produces the best sludge floc with the least amount of chemical—the following studies relate particularly to this compound. Other coagulating aids and substitutes, such as lime, chlorinated copperas, and ferric and aluminum sulfate, are also considered.

Review of Established Pertinent Facts

Experience has already taught the following pertinent facts about sludges and their liquid requirements for chemical coagulants:

(*a*) The major fraction of all sludges is the water that must be strained from the solids by coagulation and filtration;

(*b*) The minor fraction of sludge solids consists of organic and inorganic matter, which are expressed approximately in terms of volatile matter and ash by the combustion of the dried sludge;

(*c*) The volume of the major fraction of water depends primarily on the specific gravity of the solids, which in turn depends on the ratio of organic to mineral matter in the sludge solids;

(*d*) The organic fraction of sludge solids is destroyed partly by bacterial action during storage of freshly collected raw solids, and it is destroyed materially in controlled sludge digestion. This action not only reduces the ratio of organic to mineral matter, but it also increases both the water fraction and the solute concentration of such decomposition products as the bicarbonates of ammonium and calcium, in the water fraction. Instances are known in which the bicarbonates present in fresh raw sludges increase almost one hundred times through digestion. Measured in terms of percentage on solids present, an increase of ten to twenty times is not uncommon.

Sludge-Water Demand for Ferric Chloride

Studies of the effects of elutriating decomposing sludges by the writer,[2] C. E. Keefer, M. Am. Soc. C. E., and Herman Kratz, Jr.,[3] at Baltimore, Md., and P. D. McNamee[4] at Washington, D. C., have disclosed and confirmed the fact that the water associated with the solids in a sludge has a specific chemical demand of its own which may be regarded as distinct from that of the solids. This water or "liquid demand" is largely a function of items (*c*) and (*d*).

The addition of ferric and aluminum salt solutions to solutions of ammonium and calcium bicarbonates causes definite chemical reactions; that is, the liberation of carbon dioxide gas and the formation of precipitates of ferric and aluminum hydroxide. According to the combining weight relations of this reaction for ferric chloride:

$$\underset{(162.21)}{FeCl_3} + \underset{(3 \times 79.06)}{3NH_4HCO_3} \rightleftharpoons \underset{(106.86)}{Fe(OH)_3} + \underset{(3 \times 53.5)}{3NH_4Cl} + \underset{(3 \times 44.01)}{3CO_2} \quad \ldots (A)$$

and

$$\underset{(2 \times 162.21)}{2FeCl_3} + \underset{(3 \times 162.12)}{3Ca(HCO_3)_2} \rightleftharpoons \underset{(2 \times 106.86)}{2Fe(OH)_3} + \underset{(3 \times 111)}{3CaCl_2} + \underset{(6 \times 44.01)}{6CO_2} \quad \ldots (B)$$

In these reactions one need be concerned only with the numerical weight relation of the reaction between the ferric chloride and bicarbonate ion. As the standard method of measuring bicarbonate alkalinity (HCO_3^-) is in parts per million of calcium carbonate, $CaCO_3$, the weight relationship between $FeCl_3$ and HCO_3^- shifts to that between parts per million of calcium carbonate, and parts per million of ferric chloride. In reactions *A* and *B*, $2FeCl_3 = 6HCO_3^- = 3CaCO_3$. Therefore, 1 ppm $CaCO_3 = \frac{2}{3} \times \frac{162.21}{100} FeCl_3$ = 1.08 ppm $FeCl_3$.

As it is common practice in sludge conditioning to measure the chemicals used in terms of sludge solids—namely, as the percentage of sludge solids—the alkalinity should be measured in the same manner. The percentage alkalinity should then be multiplied by the foregoing constant 1.08 to arrive at the liquid demand for ferric chloride in terms of the solids.

Let Y_1 = alkalinity expressed as percentage of solids, then

$$Y_1 = \frac{\text{Percentage of sludge water} \times \text{ppm alkalinity} \times 0.0001}{\text{Percentage of sludge solids}} \quad \ldots (1)$$

Let X_1 = percentage of ferric chloride required on sludge solids to complete the chemical reactions with this alkalinity. Then,

$$X_1 = 1.08\, Y_1 \quad \ldots (2a)$$

or

$$Y_1 = 0.926\, X_1 \quad \ldots (2b)$$

[2] "Adsorption and Flocculation as Applied to Sewage Sludges," by A. L. Genter, *Sewage Works Journal*, July, 1934, p. 689.

[3] "The Vacuum Filtration of Elutriated Sludge," by C. E. Keefer and Herman Kratz, Jr., *ibid.*, September, 1934, p. 845.

[4] "Filtration Tests with Elutriated and Unelutriated Digested Solids," by Paul D. McNamee, *ibid.*, September, 1939, p. 764.

Eqs. 2 are the simplest possible formulas of a straight line, expressing the fact that the sludge liquid demand for ferric chloride is a straight-line function of the percentage of alkalinity present on sludge solids.

Solids Demand for Ferric Chloride

As some of the bicarbonates and other solute reagents are undoubtedly trapped within the spongy or capillary porous structure of the wetted sludge solids it is logical to assume that this solid water complex likewise has a chemical affinity or demand for ferric chloride.[5] This is conveniently termed the "solids demand" in order to distinguish it from that of the large fraction of free water in which the solids are suspended. The latter liquid demand must be satisfied largely before additional ferric chloride can reach the solid complexes.

In view of what has already been stated about the influence of the organic-to-mineral ratio of the sludge solids on sludge moisture, the solids demand for ferric chloride should be subject, in some manner, to the same influence. If the solids were entirely mineral, they would require little or no coagulation for vacuum filtration.

Basic Data Used in Determining Solids Demand

If the foregoing statements are fundamentally reliable, it should be possible to estimate approximately the combined liquid and solids demands for ferric chloride with such simple plant data as percentages of solids, water, and volatile matter in a sludge and the parts per million of alkalinity in the sludge water. In plant practice this involves not only reliable statistical averages in sampling and analytical methods but also careful vacuum filter operation. Careful filter operation infers: (*a*) Regulating the ferric chloride dosage so that this chemical is not wasted; (*b*) operating with a filter speed and drum submergence as required by the conditioned sludge; and (*c*) operating with a minimum of filtrate resistance in the filter cloth and in all drainage channels under this cloth. Items (*a*) and (*b*) are quite important in avoiding errors due to frequent faulty addition of any coagulant to a sludge, as well as overflowing sludge that has been conditioned either from a conditioning tank or from the filter itself. LeRoy W. Van Kleeck[6] has emphasized tersely the importance of item (*c*).

With these objectives and items in mind all data presented in Table 1 are purposely selected in order to establish a fairly reliable starting point for evolving a factor for the sludge solids demand. The other incidental objective was to choose data representing geographic variations in plant location, and variations in types of sludges, sludge moistures, alkalinities, volatile content, and ferric doses. The number of plants collecting sludge alkalinity data is not very large and in some plants the tabulation of such data does not extend over several years.

The largest raw sludge filtration plant publishing complete data in the United States is the Minneapolis-St. Paul (Minn.) plant (line 1, Table 1). It

[5] "Principles and Factors Influencing Vacuum Filtration of Sludge," by A. L. Genter, *Sewage Works Journal*, November, 1941, p. 1182.

[6] *Ibid.*, p. 1207.

is also outstanding in careful filter operation. The essential assay figures for this plant are taken from analytical results for 1941 by George J. Schroepfer,[7] Assoc. M. Am. Soc. C. E. As both lime and ferric chloride are used at this plant, the equivalent ferric dose (Col. 9, Table 1) is determined by the method developed in this paper under the heading, "Other Coagulants: Lime and Ferric Chloride."

TABLE 1.—TYPES AND ASSAYS OF SLUDGES CHOSEN TO DETERMINE A FACTOR FOR SLUDGE SOLIDS DEMAND

Line	Type of sludge	Alkalinity (ppm)	Sludge Assays (%)				Alkalinity[a] / Solids	Alkalinity on solids[b]	$FeCl_3$ (%)			
			Water	Solids	Volatile	Ash			Total dose	Used for alkalinity	Left for coagulation	Total dose[c]
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
1	Primary thickened[d]	260	90.65	9.35	62.1	37.9	0.00252	0.252	2.5	0.27	2.23	2.9
	Digested Elutriated—											
2	Hartford, Conn.	150	88.6	11.4	41.1	58.9	0.00117	0.117	1.26	0.13	1.13	1.25
3	Springfield, Mass.	235	88.6	11.4	48.3	51.7	0.00183	0.183	1.63	0.2	1.43	1.69
4	Winnipeg, Man., Canada	1,000	90.0	10.0	50.5	49.5	0.009	0.90	2.6	0.97	1.63	2.63
5	San Francisco, Calif.	470	96.85	3.15	63.5	36.5	0.0145	1.45	4.3	1.6	2.7	4.3
	Digested Waste—											
6	Activated	3,500	96.5	3.5	60.0	40.0	0.0965	9.65	13.0	10.42	2.58	12.8
7	Elutriated	500	94.2	5.8	57.0	43.0	0.0081	0.81	3.0	0.87	2.13	3.0
8	Waste activated thickened	140	98.0	2.0	80.0	20.0	0.007	0.69	7.3	0.74	6.56	7.14

[a] Ratio by weight. [b] Percentage of alkalinity on solids. [c] Computed. [d] Minneapolis-St. Paul plant.

Whereas the Minneapolis-St. Paul data are outstanding in effecting coagulant economies resulting from careful operations in the filtration of primary raw sludge, the writer regards Springfield, Mass. (line 3, Table 1) as the pacemaker in formulating similar methods of filtering elutriated digested primary sludge for populations exceeding 100,000. Line 3, Table 1, gives data for the entire year of 1941 (the first year of operation at Springfield).

The data of line 2, Table 1, were selected primarily for comparison with the data in line 3 and do not represent the entire corresponding average for the year at Hartford, Conn. A particular month was selected during which the averages showed identical solids in the case of Hartford and the year's average at Springfield (Col. 4, Table 1), but showed lower alkalinity, volatile content, and ferric dose (Cols. 2, 5, and 9, Table 1) at Hartford. It was assumed that the lower values for alkalinity and volatile percentage with identical solids content at Hartford accounted for the lower ferric dose during the particular month and should, therefore, provide extreme low points for such values in plotting the graphic results for Fig. 1.

The averages in lines 4 and 5, Table 1, like lines 1 and 3, are the averages for the entire year of 1941 at Winnipeg, Manitoba, Canada, and the Richmond-Sunset treatment plant in San Francisco, Calif. Particular care is used in

[7] *Proceedings*, Am. Soc. C. E., January, 1943, p. 56 (and private correspondence).

conditioning the sludge at Winnipeg, and K. Fraschina[8] has emphasized studies undertaken to improve careful operation at the Richmond-Sunset plant.

In a way the sludge filtered at Richmond-Sunset is remarkably different from the others. Operation of this plant involves mechanical preflocculation

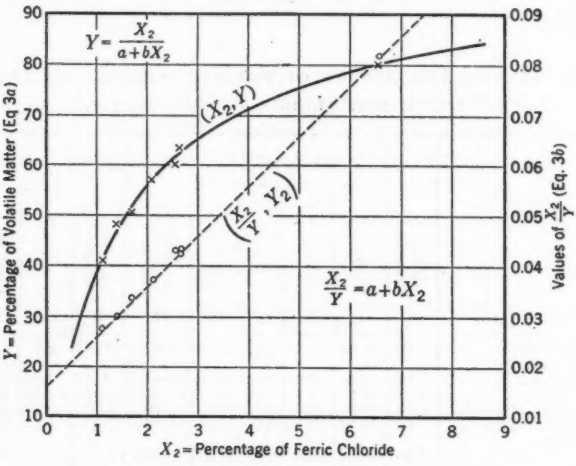


FIG. 1.—VOLATILE-FERRIC CHLORIDE DOSING CURVES

of the solids suspended in the sewage, single-stage digestion with about 62% destruction of 82% raw volatile matter, and elutriation of both supernatant and bottom solids from the digestion tank.

Line 6, Table 1, is chosen to represent about the most difficult type of sludge to coagulate and filter, either with ferric alone or ferric and lime combined—namely, digested mixed primary and waste activated sludge. Such sludges are usually low in solids and high in alkalinity. The sludge shown in line 6 was conditioned with lime and ferric chloride at Ann Arbor, Mich., in 1940 just prior to making some plant-scale elutriation tests. The combined price of the lime and ferric used amounted to the equivalent of about 13% ferric chloride alone, which sum is entered as Col. 9 for this sludge. A large quantity of this sludge was elutriated, conditioned, and filtered with the results shown in line 7, Table 1. Although the data shown are experimental, the writer regards them as characteristic.

Ferric chloride is used alone in conditioning waste activated sludges. However, in larger plants which condition and filter such sludges, alkalinity data are either entirely or largely missing because the ferric dose is primarily regulated by the hydrogen-ion concentration of the conditioned sludge. This is generally about pH 4.3. As such sludges are notably high in water content, considerably more ferric may be used in conditioning them to a pH 4.3 than to a pH 4.9 or pH 5, especially with automatic control of the dosing. There-

[8] "Laboratory Control of the Operation of the Richmond-Sunset Sewage Treatment Plant," by K. Fraschina, *Sewage Works Journal*, January, 1940, pp. 95–96.

fore, purely experimental data had to be used for such a sludge in line 8, Table 1. A sludge of high volatile content and low alkalinity was purposely selected to provide an extreme high point for graphic plotting of the influence of volatile matter on conditioning. The sludge was conditioned to about pH 4.9. Like waste activated sludge, the elutriated digested sludges in lines 2, 3, 4, 5, and 7, Table 1, are all conditioned with ferric chloride alone.

In Table 1 it will be noted that the variations in volatile contents of the listed sludges are greater than 100%, the variations in solids and alkalinity are 600% and 2,400%, respectively, and the ferric doses vary from about 1.3 to 10.0 times as much—namely, 13%.

Factor for Solids Demand Evolved

From the selected data in Table 1 the percentage of ferric chloride used for coagulation of the wetted sludge solids and incidental reactions should be determined with fair practical accuracy by deducting the percentage used for the alkalinity reaction, determined by Eq. 2*a* from the total ferric chloride used. In Table 1 this has been done in Cols. 9, 10, and 11. Col. 11 then shows the percentage ferric chloride on solids used for the volatile-solid complex, which may also be termed the ferric demand of the wetted solids themselves.

When the values of this demand in ferric chloride are plotted on the abscissa axis X_2 (see Fig. 1) against the percentages of volatile matter in the respective sludges listed as ordinate (Y-axis) values, a remarkably regular curve results. As shown in Fig. 1, this is a hyperbolic curve of the type

$$Y = \frac{X_2}{a + b\,X_2} \qquad (3a)$$

which converts to a straight-line equation

$$\frac{X_2}{Y} = a + b\,X_2 \qquad (3b)$$

Plotting the values for X_2 (Col. 11, Table 1) against these values divided by the respective volatile percentages Y (Col. 5, Table 1), as shown in the right-hand side of Fig. 1, results in a straight line. Using the simple method of averages on the six sludges dosed only with ferric chloride, the two constants calculate closely to $b = 0.01$ and $a = 0.016$.

Therefore the straight-line equation for solids demand is

$$\frac{X_2}{Y} = 0.016 + 0.01\,X_2 \qquad (4a)$$

which, multiplied by 100 to eliminate the constant 0.01, converts to $X_2 = \frac{1.6\,Y}{100 - Y}$. As Y is the percentage of volatile matter in the sludge, then $100 - Y$ is nothing but the percentage of ash, which simplifies Eq. 4*a* to

$$X_2 = 1.6\,\frac{\text{Percentage volatile}}{\text{Percentage ash}} \qquad (4b)$$

Eq. 4*b* is another straight-line equation for the solids demand.

The total ferric chloride dose, X, as percentage on solids is therefore the sum of the two straight-line equations (Eqs. 2*a* and 4*b*); that is,

$$X = 1.08\ Y_1 + 1.6\ Y_2 \quad \ldots\ldots (5)$$

in which Y_1 is the percentage of alkalinity on sludge solids according to Eq. 1 and Y_2 is the ratio of volatile matter to ash in the sludge.

Col. 12, Table 1, shows the doses calculated according to this equation for the various sludges listed.

Facts Revealed by Eq. 5

As a result of the reagents involved, Eq. 5 reveals a definite chemical influence in sludge conditioning. As it is primarily based on alkalinity assays and water-to-solids ratios this formula measures a definite reaction with ferrichloride and the alkalinity of the total water—namely, the free water and the residual within the sludge solid capillary structure. Such chemical reactions obviously involve basic and experimental constants.

In order to relate the ferric dose required for alkalinity to the sludge solids present in the first part of Eq. 5, it is necessary to use the ratio of water to solids. This ratio is evidently determined by the ratio of volatile matter to ash in the sludge solids in the second part of Eq. 5. This is thoroughly logical and may be regarded as a confirmation of the empirical law relating to the Mohlman sludge index, used for measuring the concentration of solids from mixed liquors, which states that this index correlates closely and varies inversely as the ash content of the sludge.[9] In reality all sludge solids complexes suspended in water may be said to have a sludge index.

Therefore it appears that, at present, the constant 1.6 is the only empirical factor or constant in the equation. It should be emphasized that the value of this constant may increase beyond 1.6 with dosing increments purposely used to increase filter yields or to produce minimum cake moistures for the heat drying of some sludges. Instances may be cited in which the constant is practically double the value herein evolved. Conversely there are instances of very low filter speeds (8 min per revolution), heavy sludge, and low dosage, for which this constant becomes practically the same as the first constant, 1.08.

For general mathematical purposes, Eq. 5, when ferric chloride and alkalinity assays are used, evidently becomes

$$X = 1.08\ Y_1 + K\ Y_2 \quad \ldots\ldots (6)$$

in which K is an empirical constant which probably varies from the value of the first constant (1.08) to approximately three times this value. From the selected data for careful dosing it is about 1.5 times the minimum value—namely, 1.6.

The range of values for this constant, below 1.6, may be demonstrated to be representative of partial or incomplete ferric dosing of heavy sludges of relatively low alkalinity. The less complete the alkalinity and coagulation reactions are the slower will be the resulting filtration rates. This is quite evident from examination of the numerous filter yield graphs that have been

[9] "Problems and Trends in Activated Sludge Practice," by Robert T. Regester, *Transactions*, Am. Soc. C. E., Vol. 106 (1941), p. 176.

published since 1933. Mr. Schroepfer[10] clearly shows this effect on raw sludge dosed with progressive increments of lime and ferric. From the data in Table 1, Item 1 (Cols. 5, 6, and 11), this constant for the 1941 Minneapolis-St. Paul data is computed to be about 1.35 when using the price equivalent of ferric alone. This certainly indicates partial, and therefore most careful, dosing.

Particle Size.—In revealing reactions largely of a chemical nature, Eqs. 5 and 6 ignore particle size—that is, the presence of finely dispersed colloids in any sludge. This is entirely logical. It would be unreasonable to assume that the various sludges listed in Table 1 contained particles or sludge aggregates of the same size.

Very finely divided solids in sewage will settle to a thin sludge having a high water-solid ratio. With bicarbonates present in the water this high ratio automatically increases ferric demand in terms of the solids. An outstanding example of this statement will be found in Mr. McNamee's filtration tests on the liquor associated with digested sludge.[4] Such supernatant liquor is rich in finely divided solids. In Mr. McNamee's paper[4] the total solid content was reported as 0.4% dispersed in a very high water-to-solid ratio (249 to 1) with a total alkalinity of 3,630 ppm in the water, and a volatile-ash ratio of 1.26. On the basis of Eq. 5, the liquid alkalinity demand for ferric chloride was 98% and the solids demand only about 2%, making a total of 100% ferric dose on solids present. This is remarkably close to the total optimum requirements (105%) for completed alkalinity and coagulation reactions established by properly analyzing the McNamee filtration data. With 0.38% actual sludge solids (which is practically the 0.4% total solids listed by Mr. McNamee), the calculated values total 105% ferric chloride. This is regarded as a rigid test for the validity of Eq. 5.

Floc Size.—This is an important factor in obtaining good filter yields at careful dosing rates and will be briefly dealt with hereinafter.

Eq. 5 Applied to Other Experimental Results Not Tabulated

Usually the best operating control for conditioning sludges is made by frequent laboratory checks with Buechner funnel filters and about 200 g sludge samples of the dosed sludge going to the filter. The laboratory-tested samples should dry to a vacuum break in less time than that required to complete a normal filter drum revolution. More informative, and therefore more reliable, checks are made on sludge samples dosed in the laboratory over various ranges of ferric chloride additions, and the Buechner filter rates in grams of dry solids per minute plotted against the percentages of ferric added are as shown in Fig. 2.

Sludges dosed for actual filter practice require more coagulant to produce sufficiently dry filter cake in a given time than do sludges tested in the laboratory. What may be a dose optimum in laboratory testing will be less than optimum in field practice. There are numerous logical reasons for this: Laboratory testing is essentially small batch filtration, usually done more carefully and at a higher vacuum than in field practice; sludge drainage is directly downward instead of upward against a revolving drum surface and therefore without

[10] *Proceedings*, Am. Soc. C. E., January, 1943, Table 12, p. 63.

cake scouring; and the dosed sample is not as long in reaching the filter with the consequence that there is no destruction of the coarsened drainable floc structure produced by either partial or complete coagulation. Therefore, laboratory filtration rates for 4 or 5 min of drying time may be pushed to

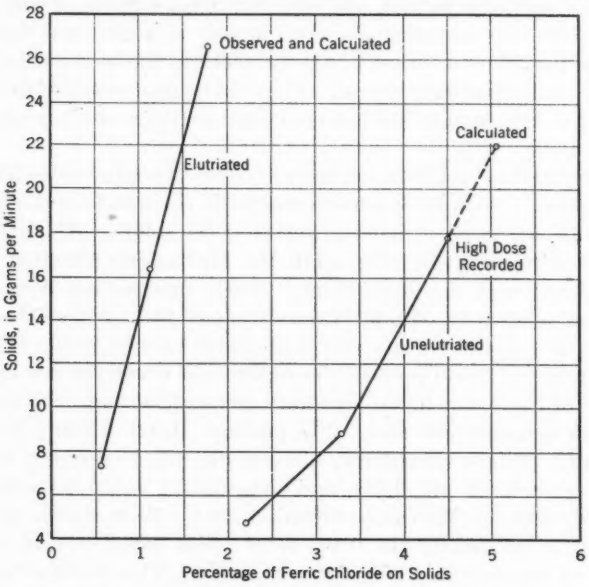


FIG. 2.—FILTER YIELD, FERRIC CHLORIDE DOSING CURVES (MCNAMEE TESTS)

considerably lower partial ferric doses than in the field, where coarse floc may be initially produced but cannot always be maintained for many reasons.[8,11,12,13]

As Eq. 5 accounts for complete alkalinity reaction and probably complete volatile coagulation, the confirmation of this equation by Buechner filter tests has two requirements—the calculated dose should be somewhere near the upper end of the plotted graphs (that is, near the optimum dry solid yield rate), and it should fall directly on such graphs. Obviously reliable sampling, sludge assays, testing technique, and the average of a number of tests are best for this purpose.

Mr. McNamee's laboratory studies of digested supernatant were adopted as the initial test of the validity of Eq. 5. In his published work,[4] the course of alkalinity removal from digested sludge through elutriation and the effects of this removal on sludge conditioning were followed for the first time. His tables showing seventy-two coagulation and filtration tests on unelutriated and elutriated sludges were next used in preparing graphical data for Fig. 2 and as a test of the validity of Eq. 5. Here the graph points of average low, medium,

[11] "Principles and Factors Influencing Vacuum Filtration of Sludge," by A. L. Genter, *Sewage Works Journal*, November, 1941, pp. 1187, 1189, and 1192.
[12] "Effect of Stirring of Sludge on Vacuum Filtration," by C. E. Keefer and Herman Kratz, Jr., *ibid.*, September, 1937, p. 743.
[13] "The Richmond-Sunset Sewage Treatment Plant," by B. Benas, *ibid.*, January, 1940, p. 93.

and high doses used are plotted against average solids yields for the thirty-six filtration tests on straight digested sludge and the thirty-six tests on the elutriated sludges. At practically completed alkalinity and coagulation reactions the equation was found valid, and the dose calculated from the average assays of the unelutriated and elutriated sludges fell on the respective graphs near the maximum recorded yield points.

The essential analyses for the digested sludge at Cranston, R. I., before and after elutriation, are shown in Fig. 3. Comparisons of the computed doses are

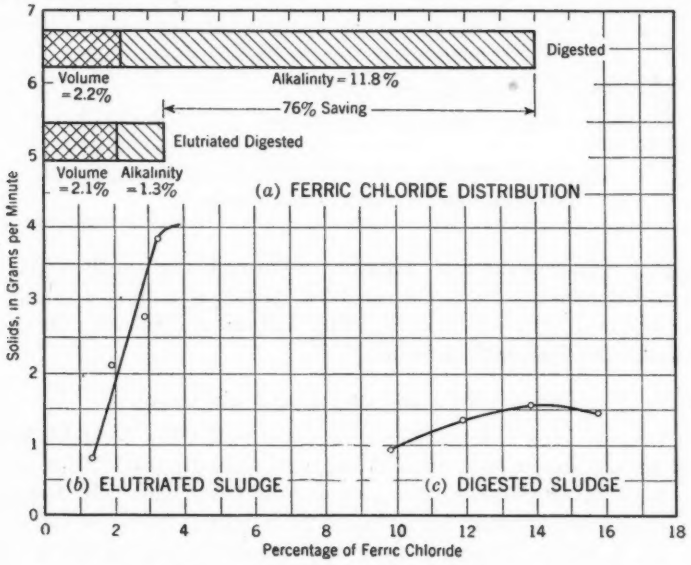


FIG. 3.—FILTER YIELD, FERRIC CHLORIDE DOSING CURVES, CRANSTON, R. I.

presented in Table 2. This new plant[14,15] produces digested mixed primary and waste activated sludge, which for obvious reasons has a high ferric chloride demand. In Fig. 3 and Table 2 the optimum graphed doses are remarkably close to the calculated doses.

Instances of laboratory tests on other sludges may be cited. In some of these the calculated dose is somewhat less than the recorded optimum, indicating a somewhat higher optimum constant than 1.6. However, in all cases thus far recorded the calculated doses fall on the recorded graphs and are therefore regarded as distinct point indexes of near optimal doses in laboratory testing and a reasonable pacemaking low dose in actual practice.

Similarly, in the exhaustive field tests made and recorded by Messrs. Keefer and Kratz,[3] Eq. 5 demonstrates clearly, through point indexes, the relative positions of the various graphs (similar to Figs. 2 and 3) for all sludges dosed

[14] "Up-to-the-Minute Cranston, R. I., Sewage Disposal Plant and Sewerage System," by Ralph W. Horne, *Public Works*, May, 1943, p. 20.

[15] "Compact Sewage Treatment Works Units at Cranston, R. I.," by Ralph W. Horne, *Civil Engineering*, September, 1943, p. 417.

with ferric chloride. These field tests, made in 1933 and 1934, were run in a manner very similar to laboratory tests—that is, on different batches of large samples of various sludges within various dosing ranges. Here the effects of various concentrations of dissolved ammoniacal nitrogen in parts per million

TABLE 2.—COMPUTATION OF FERRIC CHLORIDE DISTRIBUTION IN ELUTRIATED AND DIGESTED SLUDGE; CRANSTON, R. I.

Description	Elutriated (Fig. 3(*b*))	Digested (Fig. 3(*c*))
Alkalinity (parts per million)	300	2,700
Analysis (Percentage by Weight[a])—		
Volatile matter	57	58
Mineral matter	43	42
Computed Values of $FeCl_3$—		
X_1 (by Eq. 2*a*)	1.3	11.8
X_2 (ratio of volatile to mineral, multiplied by 1.6) (Eq. 4*b*)	2.1	2.2
Total, by computation	3.4	14.0
Total, recorded	3.3	13.9

[a] Water 97.6% and solids 2.4% in both sludges.

on coagulant requirements were followed. Through the known ratio of ammonium to total bicarbonates the recorded parts per million of ammoniacal nitrogen produced values for Eq. 5. The various graphs were then located at points near the dose optima recorded.

EQS. 5 AND 6 AS AN OPERATOR'S GUIDE

From the foregoing observations it is evident that Eq. 5 establishes more of a point than a line on such ferric dosing and yield graphs. Other K-values in Eq. 6 may be established from proper laboratory and field testing. In practice, if the actual average ferric dose is materially in excess of values calculated by Eq. 5, careful Buechner filter tests should be run in the laboratory. Here this equation will be an indicating guide to the approximate dose optimum necessary.

If such laboratory tests indicate the approximate validity of the equation and if plant practice on the same sludge does not, the operator has good evidence that chemical is being wasted somewhere in processing the conditioned sludge. The chief sources of this trouble have been cited.[5,8,12,13]

If laboratory tests denote that much higher doses are required than are indicated by Eq. 5, this should necessitate the graphical plotting of the average of several laboratory tests to determine if the constant K in Eq. 6 should be changed to a value materially higher than 1.6.

Exceptions to Eq. 5.—Such a change in the value of K may be due to the logical exceptions found in sludges containing undue amounts of ferric reducing agents or other coagulant-consuming compounds resulting from industrial wastes. In some cases such waste products may be beneficial and in others distinctly inimical to good coagulant economy. In sludges containing mineral oil wastes, cutting oil with soaps, etc., various emulsions and ferric consuming smears (ferric soaps) can result, thus increasing the ferric demand while adversely influencing dry cake yields. Such atypical conditions do not invalidate

Eq. 5. Some of these complexes may be accounted for through proper laboratory tests and the constant K changed to fit local conditions; or the complexes can be largely eliminated by other methods.

Distribution of Coagulant Requirements in Various Types of Sludges

In Fig. 3(*c*) the distribution of the portions of the total ferric dose for liquid and solids demand is shown graphically for the sludges tested in the laboratory at Cranston. With the unelutriated sludge, 84.3% of the total dose is used to offset alkalinity alone, leaving only about 16% for the actual coagulation intended. As elutriation displaced most of the alkalinity factor and a small portion of the volatile matter, most of the ferric dose remained for solids demand.

A similar graphical method could be used on the various sludges listed in Table 1. However, such a procedure would not depict the quantity of ferric chloride related to the various quantities of sludge collected and processed in different types of plants.

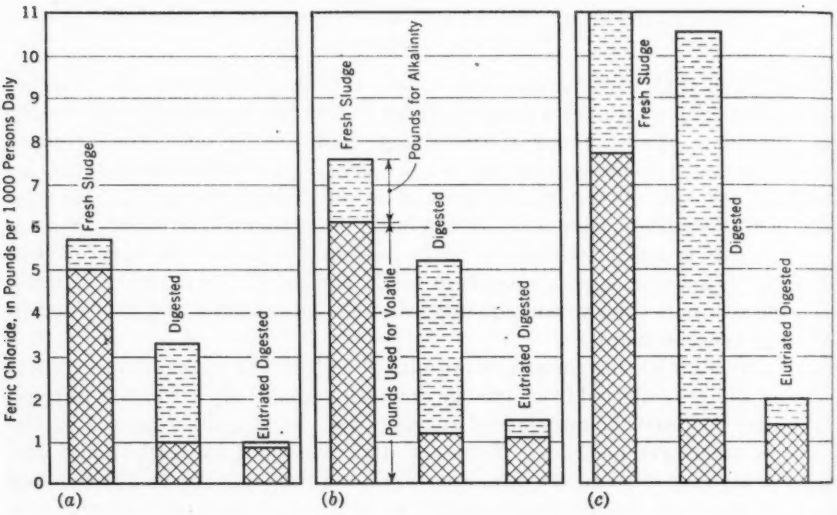
A more illuminating comparison results when the data are limited to various types of sludges collected from average domestic sewage only. For this purpose the writer has used the amounts of average fresh sludge solids removed by the different treatment methods. Tables of data presented by Karl Imhoff and Gordon M. Fair,[16] Members, Am. Soc. C. E., were extended to coagulation and vacuum filtration and the results are given in Table 3, the ferric chloride distribution for the liquid and solids demand being shown in pounds per 1,000 persons daily.

In calculating the pounds of digested solids on this basis, the writer has assumed in all cases a reduction of 65% of the fresh solid volatile in digestion, and 66% in digestion followed by elutriation, as the latter process removes some volatile and further concentrates the sludge. There is always a loss of some solids through digester supernatant withdrawals, elutriation, and vacuum filtration. Consequently, in Table 3 the average digested and elutriated solids reaching the filters will be found to be somewhat higher than in practice. A good example of this statement is to be found in the 1942 results for Springfield. About 130 lb per 1,000 persons daily of primary solids, of 74% volatile matter, were collected. The percentages of volatile matter reduction in digestion, and through digestion and elutriation, were 65% and 66.3%. The removal efficiency of suspended solids was about 60%. The digested sludge contained 8.5%, and the elutriated sludge 12% solids compared to 8% and 10% shown in Table 3. The percentage of ferric chloride consumption was identical with that computed in Table 3. However, due to loss of solids in processing digester supernatant, elutriate, and filtrate, the actual ferric consumption was about 0.7 lb per 1,000 persons daily instead of 1 lb shown in Table 3.

The percentage solids (Col. 6, Table 3) and alkalinities (Col. 5, Table 3) in the various sludges are taken from conservative averages collected by the writer. In storing and concentrating primary sludge comprising 72% volatile

[16] "Sewage Treatment," by Karl Imhoff and Gordon M. Fair, John Wiley & Sons, Inc., New York, N. Y., 1940, Tables 11 and 12, pp. 185–189.

TABLE 3.—COMPUTATIONS OF THE QUANTITY AND NATURE OF DOMESTIC SLUDGE AND FERRIC CHLORIDE REQUIRED FOR VACUUM FILTRATION



No.	Treatment process	Solids[a]	Percentages		Alkalinity (ppm)	Percentage of solids	Ferric Chloride Used[a]		
			Volatile	Ash			For alkalinity	For volatile	Total
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
	(*a*) PLAIN SEDIMENTATION (SEE FIG. (*a*), ABOVE)								
1	Solids removed..	119	72	28	400	6	0.7	5.0	5.7
	Remainder:								
2	Digested.....	64	47.5	52.5	3,000	8	2.3	1.0	3.3
3	Elutriated[b]....	62	46.5	53.5	220	10	0.1	0.9	1.0
	(*b*) TRICKLING FILTERS AND SEDIMENTATION (SEE FIG. (*b*) ABOVE)								
4	Solids removed..	148	72	28	400	6	1.5	6.1	7.6
	Remainder:								
5	Digested.....	79	47.5	52.5	3,000	6	4.0	1.2	5.2
6	Elutriated[c]....	77	46.5	53.5	330	7	0.4	1.1	1.5
	(*c*) ACTIVATED AND SEDIMENTATION (SEE FIG. (*c*) ABOVE)								
7	Solids removed..	187	72	28	500	3	3.3	7.7	11.0
	Remainder:								
8	Digested.....	100	47.5	52.5	3,000	3.5	9.0	1.5	10.5
9	Elutriated[c]....	97	46.5	53.5	330	4.5	0.7	1.3	2.0

[a] In pounds per thousand persons daily. [b] Elutriated with water of 100 ppm alkalinity at an elutriation ratio R of 4 to 1 (countercurrent). [c] Elutriated with plant effluent of 200 ppm alkalinity at an elutriation ratio R of 4 to 1 (countercurrent).

matter and its mixtures with secondary sludges for filtration, the alkalinity (Col. 5, Table 3) can easily exceed the listed values, especially in warm weather. Obviously such sludges of high alkalinity can be benefited by proper elutriation. However, such treatment has not been included in Table 3, since it is not typical practice in the United States.

Mechanically flocculated sludges like those involved at Richmond-Sunset and the new San Diego, Calif., plant are quite light and will probably be found to be somewhat like the graph for Table 3(*c*), with activated sludge when properly digested and thoroughly elutriated.

Although sludges collected from combined sewerage systems may double in quantity the amounts shown in Col. 2, Table 3, the total coagulant and its distribution will not double. In fact for fresh plain sedimentation the Col. 9, Item 1 value may be but little more. This is due to the increase of mineral and decrease in volatile matter in such sludges. This is quite evident at Minneapolis-St. Paul where the percentage of volatile matter in the concentrated raw sludge is actually less than that of the digested sludge at Richmond-Sunset (see Table 1) with the result that the primary sludge at the former plant is very heavy and very easy to condition. The 1941 Minneapolis-St. Paul figure for Col. 2, Item 1 of Table 3, was about double the listed value, that is, 246 lb. As a result of the low dose requirements (see Table 1) the value in Col. 9, Item 1 of Table 3, is 6.15 lb, which is only 8% more than the 5.7 lb listed. If this particular sludge were digested and elutriated the values in Col. 9 for Items 2 and 3 would increase about 60% and become more like the Col. 9 values for Items 5 and 6.

The bar graphs in Table 3 supply illuminating evidence of where ferric chloride goes for the various types of sludges. Where the percentage of alkalinity on solids is relatively low as in the primary sludge, its mixtures, and the elutriated sludges, most of the ferric is used for the solid demand. The reverse is the case with the unelutriated digested sludges in which, despite good mineralization of the solids and lowered volatile-to-ash ratio, the high percentage of alkalinity makes this factor predominant in coagulant requirements.

The graph in Table 3(*c*) certainly indicates that there is little if anything gained in using vacuum filters on digested mixed primary and waste activated sludges of relatively high volatile content without the benefits of elutriation. Due to the light solid content of such sludges it can be demonstrated in several instances known to the writer that more ferric chloride or equivalent ferric chloride and lime per 1,000 persons is used on the digested mixture than would be used on the fresh undigested mixture. For example, by changing the percentage of solids in this digested sludge (Col. 6, Item 8, Table 3) to 2.5, the alkalinity factor (Col. 7) becomes 12.6 lb, which, together with the same volatile factor, totals more than 14 lb of ferric chloride (Col. 9) per 1,000 persons daily. This is 27% greater than the chemical demand shown for the fresh sludge mixture. In this example of digested sludge, the alkalinity demand is almost 90% of the total requirements. It is certainly faulty practice to coagulate such sludges and dewater them in vacuum filters without elutriation. If elutriation is not to be used, it is far better engineering to provide sand-bed filtration.

Volatile Matter Destruction as a Measure of Chemical Saving

The percentage of volatile matter destroyed is identical with the percentage reduction in the volatile-ash ratio through the digestion of any sludge. Although the true significance of this statement has been overlooked, Eqs. 5 and 6 now lend it importance. As the reduction in the volatile-ash ratio of a sludge means a corresponding reduction in ferric chloride demand for this ratio, the reduction in volatile matter through digestion is a direct measure of ferric chloride saved per pound of original volatile matter, providing the resulting alkalinity gain is removed by elutriation. However, as the reduction in ferric demand through this procedure also leaves less solids to be conditioned, the final ferric chloride demand is used on the percentage of original raw solids left. This makes the over-all percentage of ferric reduction, for the remaining volatile-ash ratio, equal to

$$R = 100 - \frac{1}{100} \times \text{Percentage of original raw solids remaining} \times \text{percentage of original volatile content remaining} \quad (7)$$

For example, in the case of Springfield, the 1941 averages (in either pounds or percentages) showed:

No.	Description	Quantity
1	Raw volatile content (69.5% of which is removed in digestion and 30.5% of which remains)	73.6
2	Ash	26.4
3	Total	100.0
4	Remainder of original volatile content (30.5% × 73.6)	22.4
5	Plus original ash	26.4
6	Total original solids remaining	48.8

Therefore, $R = 100 - \frac{48.8 \times 30.5}{100} = 85\%$ saving in ferric chloride through digestion and elutriation. This can be checked readily through methods previously shown in Eq. 4*b*. Thus, neglecting the original and final alkalinity factors, the volatile-ash, ferric chloride requirements are:

$$\text{Original, } \frac{73.6}{26.4} \times 1.6 = 4.46\%$$

$$\text{Final, } \frac{22.4}{26.4} \times 1.6 = 1.36\%$$

The original ferric chloride requirements are intended for 100 lb of raw solids, whereas the final 1.36% is used on 48.8 lb of solids remaining; namely, 0.66 lb of ferric chloride. This is 3.8 lb less than the original 4.46 lb of ferric chloride required, or 85% saving. This saving is based on keeping the alkalinity factor constant in both instances. As a matter of fact proper elutriation may add to this saving by further reducing this factor.

One particularly interesting feature to be noted about Eq. 7 is the fact

that, regardless of the ultimate value assigned to the constant, K (1.6 in this example), Eq. 7 will not be changed.

Calculations for Sludge Elutriation.—Eqs. 5, 6, and 7 reveal the genuine advantages gained by reducing the volatile-ash ratio through digesting the sludge as much as economically possible and then removing the inimical alkalinity factor by sludge elutriation. These two procedures combine to reduce the amount of sludge solids to be handled, size of equipment required for filtration and drying or incineration, annual operating charges for chemicals, power for filter hours, and maintenance, at the same time increasing filter yields and the uniformity of sludge characteristics, and eliminating odor nuisance.

The alkalinity (in parts per million) remaining in the sludge moisture after single-stage, multi-stage, and countercurrent elutriation in two tanks may be computed from the following equations, wherein D is the alkalinity in the digested or stale raw sludge water before elutriation, E is the alkalinity of the elutriated sludge, W is the alkalinity of the elutriating water, and R is the ratio of volumes of elutriating water to the volume of moisture in the sludge: For single-stage elutriation—

$$E = \frac{D + R\,W}{R + 1} \qquad (8a)$$

for n-stage elutriation—that is, repeating the elutriation with fresh water n times at the same ratio R—

$$E = \frac{D + W\,[(R + 1)^n - 1]}{(R + 1)^n} \qquad (8b)$$

and for countercurrent elutriation in two mixing and settling tanks—

$$E = \frac{D + (R^2 + R)\,W}{R^2 + R + 1} \qquad (8c)$$

If the water and sludge are both metered, to obtain the true value of R (the ratio of water to sludge moisture by volume), the metered ratio is divided by the fraction of water present in the sludge.

With the alkalinities D, E, and W determined before and after elutriation, the elutriation ratios, R, used to obtain the alkalinities, E, may be computed by solving for R in Eqs. 8: For single-stage elutriation—

$$R = \frac{D - E}{E - W} \qquad (9a)$$

for n-stage elutriation (each wash)—

$$R = \sqrt[n]{\frac{D - W}{E - W}} - 1 \qquad (9b)$$

and for countercurrent (two tanks)—

$$R = \sqrt{\frac{D - E}{E - W} + 0.25} - 0.5 \qquad (9c)$$

In Eqs. 9, to convert R to the ratio of water to sludge, multiply the estimated values of R by the fraction of water present in the sludge.

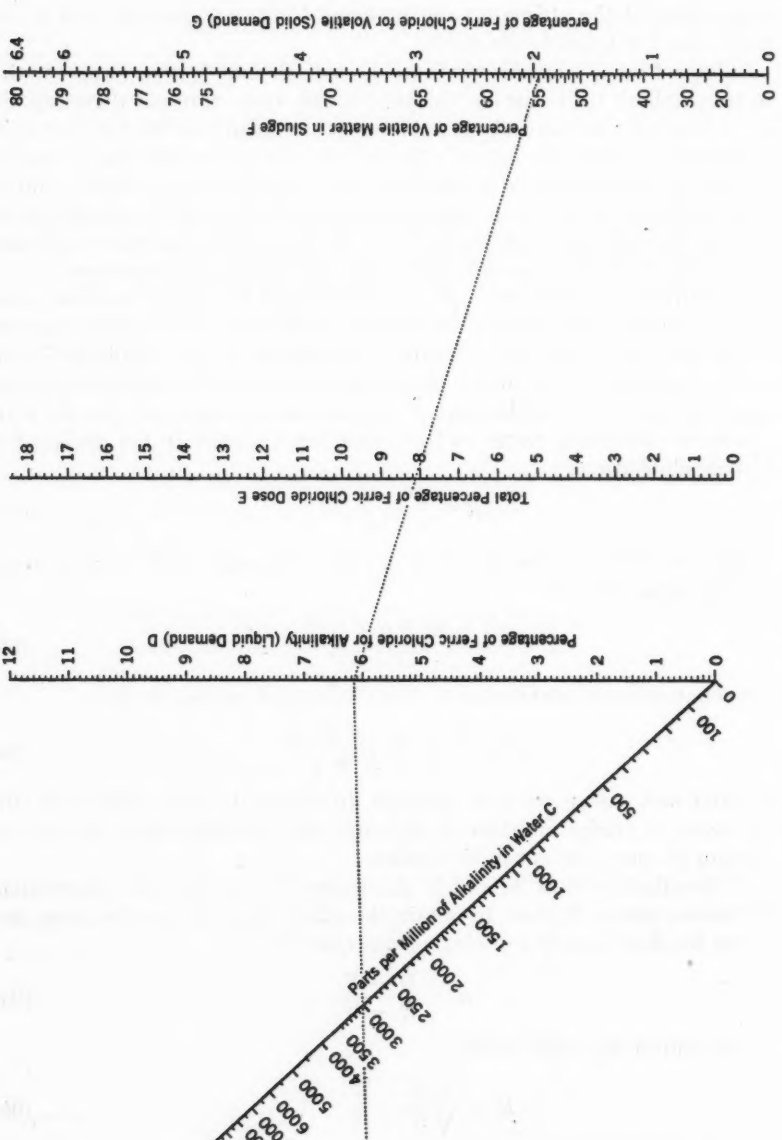

Percentage of Water in Sludge A
Percentage of Solids in Sludge B
Parts per Million of Alkalinity in Water C
Percentage of Ferric Chloride for Alkalinity (Liquid Demand) D
Total Percentage of Ferric Chloride Dose E
Percentage of Volatile Matter in Sludge F
Percentage of Ferric Chloride for Volatile (Solid Demand) G

FIG. 4.—NOMOGRAPH FOR SLUDGE DOSING; EQ. 5

Nomograph.—Fig. 4 presents a useful nomograph of Eq. 5. The N-shaped part of this graph for the alkalinity factor is scientific and therefore relatively exact, with proper sludge sampling and basic analyses.

As an example, consider a digested sludge with 5% solids, 95% water, 3,000 ppm of alkalinity, and 55% volatile content. Connect 95% on scale A, through 3,000 ppm on scale C to 6.15% on scale D. Next connect 6.15% on scale D with 55% volatile on scale F. The total ferric chloride dose (8.1%) is read at scale E. Thus, the actual alkalinity demand in this typical example is more than 6% $FeCl_3$, which is easily 75% of the total requirements. By elutriating this sludge to 300 ppm alkalinity and thickening to 8% solids the alkalinity factor becomes negligible. Fig. 4 also shows that the alkalinity factor in waste activated sludges can play a definite rôle in consuming ferric chloride; for example, at 2% solids and 400 ppm alkalinity the liquid demand is more than 2% $FeCl_3$.

Other Coagulants

Chlorinated copperas, ferric sulfate, and aluminum sulfate are being used and have been tried extensively instead of ferric chloride. More of these chemicals than ferric chloride must be used for various reasons—heavier molecular weight relations in reacting with total bicarbonates, difference in the compressible hydroxide floc formed in the presence of the sulfate radical, less purity in case of the sulfates, and greater difficulty in effectively dissolving the ferric sulfate. Therefore, these substitutes for ferric chloride must be materially cheaper than the latter in order to find successful use. They also appear to be only of value on well mineralized sludges of low alkalinity.

Chlorinated Copperas (Chemical Symbol, $FeSO_4CL$; and Molecular Weight, 187.36).—Theoretically it takes 1.16 times more of this pure coagulant than ferric chloride to complete the reactions of Eq. 5. This chemical has been used extensively at Baltimore. The indications are that the dosage approximates this ratio of about 1.2 of chlorinated copperas to 1 of ferric chloride.

Ferric Sulfate (Chemical Symbol, $Fe_2(SO_4)_3$; and Molecular Weight, 399.87).—Theoretically 1 mol of this compound reacts the same as 2 mols of ferric chloride with bicarbonates. This means that 1.233 times more of the pure sulfate salt than ferric chloride is required to complete the reactions of Eq. 5. Although this chemical is delivered in an anhydrous state and is easy to handle, as marketed commercially it is far from being as pure as ferric chloride and is much less soluble in the cold. Consequently the quantity of the commercial salt used will exceed the theoretical weight ratio to ferric chloride.

This salt as ferri-floc is being used successfully on elutriated digested sludge at Annapolis, Md., due to its price of $36.10 per ton in 1941 compared with about $99.00 per ton for anhydrous ferric chloride. Since 1941 about 2.7% ferri-floc on sludge solids was used. This is about 1.6 times as much ferri-floc as ferric chloride previously used.

Aluminum Sulfate.—The chemical symbol of pure crystal sulfate of aluminum is $Al_2(SO_4)_3 \cdot 18H_2O$; and its molecular weight is 666.42. Most commercial sulfate, however, has less water of crystallization, and a molecular weight of about 595. Theoretically it takes about 1.84 times more of this

Fig. 4.—Nomograph for Sludge Dosing; Eq. 5

commercial salt than of ferric chloride to complete the reactions of Eq. 5. In some parts of eastern United States this product, known as "paper maker's alum," is cheaper than any of the foregoing reagents. Its use on partly-elutriated and well-elutriated digested sludges presents some interesting disclosures relative to Eq. 5.

It was given extensive trials on elutriated digested sludges at Hartford in 1939, and in Winnipeg in 1940, with rather discouraging results. At Winnipeg the trials disclosed that this relatively pure alum was required in quantities 2.8 times larger than ferric chloride. Furthermore, the cake moistures were higher and the filter yields lower. These findings were confirmed at Hartford where the mineralization of the sludge was better than at Winnipeg. This was undoubtedly due to the precipitation of too much aluminum hydroxide by the bicarbonates left in the sludge. This hydroxide precipitate is exceedingly difficult to filter especially when present in excessive amounts in any sludge. What constitutes a beneficial amount of this precipitate was demonstrated in 1941 and 1942 at Springfield, where its use proved encouraging.

The same grade of alum as that used at Hartford and Winnipeg was tried at Springfield. However, in the Springfield tests the sludge was more thoroughly elutriated than at the other two plants. Therefore, the sludge alkalinity was lower. As a consequence these trials demonstrated the validity of Eq. 5 by the use of a chemical other than ferric chloride.

In 1942, during July and August, 8,023 lb of this chemical were used on 284,176 lb of sludge solids producing an average cake solid yield of 7.4 lb per sq ft hourly. The average sludge assays were 14.2% solids, 45% volatile content, and 212 ppm alkalinity. The average dose was about 2.8% alum on sludge solids. On this basis the calculated ferric chloride dose is 0.14% for alkalinity and 1.31% for the volatile factor, or a total of 1.45%. For aluminum sulfate this calculated dose would be about $1.84 \times 1.5 = 2.8\%$.

Aluminum hydroxide floc is evidently more compressible and fragile than the sludge floc produced when using ferric chloride. J. D. McDonald, engineer in charge of sewage treatment at Springfield, informs the writer that when alum was used, flash coagulation was necessary—that is, running the alum solution into the sludge stream being discharged from the bucket elevator and through baffles in the mixing chamber and then directly to the filter without any further stirring. Ferric sulfate solution is added in a similar manner at Annapolis.

Evidently, under proper conditions, crystal alum can compete with ferric salts when its pound price is less than half that of anhydrous ferric chloride, the sludge is well mineralized through digestion and then elutriated to a low alkalinity. A similar statement applies to the use of chlorinated copperas and ferric sulfate.

Lime and Ferric Chloride.—This combination is used extensively on raw and digested sludges but not on waste activated and elutriated sludges.

Eq. 5 and its nomograph, Fig. 4, are of material help in using lime and ferric chloride on those sludges where the combination may be advantageous. Lime is used with the intention of saving ferric chloride and reducing the total chemical cost per ton of sludge solids.

The combined lime and ferric requirements are just as much a function of the liquid and solids demands of any sludge as are the requirements for ferric chloride, or any of the other ferric or aluminum salts used alone. In this combination lime functions as calcium hydroxide and cannot entirely replace ferric chloride except with some sludges containing relatively large amounts of certain industrial wastes.

In a strict sense, calcium hydrate is not a sludge coagulant. Added to sludges relatively rich in bicarbonate alkalinity before ferric chloride is added, calcium hydrate precipitates the bicarbonates as calcium carbonate. The latter is an inert filter aid which prevents the liberation of carbon dioxide gas as bothersome foam when ferric chloride is added. As calcium hydroxide is only slightly soluble in water, an excess is always needed. On fairly heavy sludges, lime as CaO displaces about one half the ferric chloride dose that would be necessary if no lime were used.

Ratio of Lime to Ferric Chloride.—The importance of this ratio depends largely on the local market price of the two chemicals. This fact seems to have been overlooked in a number of installations.

The saving in the amount and cost of ferric chloride by the addition of lime depends on four essential factors: (1) The ratio of the price of ferric chloride to lime or available CaO per pound; (2) the actual ratio of the two chemicals used; (3) the minimum percentage of ferric chloride permissible with lime; and (4) the actual ferric chloride demand of the sludge. The amount of ferric chloride displaced by using lime equals the percentage of lime (L) used on sludge solids divided by the ratio (R) of the cost of ferric chloride to the cost of available CaO per pound; that is, equal to L/R. Furthermore, if the total percentage of ferric chloride used alone to produce an economic filter yield is equal to C, and the minimum percentage of ferric chloride used with lime to produce the same yield is equal to F, the economic limit to which the engineer may go in using lime is

$$C = F + \frac{L}{R} \qquad (10a)$$

or

$$R = \frac{L}{C - F} \qquad (10b)$$

Obviously $(C - F)$ is the weight or percentage, on sludge solids, of ferric chloride displaced by using lime. Therefore, the moment the ratio of percentage of lime used to maximum percentage of ferric displaced exceeds the price ratio (R), money is definitely being wasted. Nothing is gained when the two ratios are equal. In fact, it may be argued that money is being lost due to the added expense of removing lime incrustation. Consequently money can be saved only when $R > \frac{L}{C - F}$.

Increasing the value of R in Eq. 10*b* increases the range within which lime (L) can be of value in displacing ferric chloride $(C - F)$. Thus two plants in different localities may produce digested sludges having about the same total chemical demand but with ferric chloride and available lime costing $30 and

$10 per ton respectively in one plant ($R = 3$), and $72 and $12 respectively in the other plant ($R = 6$). With experimentally determined values of $C = 6$ and $F = 3$ in both plants, the lime dosage (L) in the former plant will have to be less than 9% to save money, whereas in the latter plant money may be saved at all lime doses less than 18%.

As a result of these observations, Table 4 has been compiled from the records of four of the largest and best known plants using lime and ferric

TABLE 4.—TYPICAL FERRIC CHLORIDE EQUIVALENTS TO FERRIC CHLORIDE AND LIME COMBINATIONS (PERCENTAGES)

No.	Plant	Price ratio R	CaO used L	$FeCl_3$ used F	$\frac{L}{F}$	Total $FeCl_3$		$FeCl_3$ on cake	CaO on cake	Total $FeCl_3$	
						C[a]	$R = 4.1$[b]			Observed	Computed
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
1	Minneapolis-St. Paul, Minn...	4.1	3.77	1.53	2.46	2.5	2.5	1.4	3.5	2.3	3
2	Buffalo, N. Y.....	2.82	11.2	2.9	3.8	6.9	5.6	2.4	9.7	5.8	5
	Cleveland, Ohio										
3	Westerly.....	3.24	14.0	3.0	4.67	7.3	6.4	2.4	11.1	5.8	6
4	Southerly.....	3.24	11.2	4.0	2.8	7.5	6.7	3.5	9.8	6.5	5
5	Averages.....	3.3	10.0	2.9		6.0	5.3			5.0	5.0

[a] Total percentage of ferric chloride, equivalent (symbol C). [b] Total percentage of ferric chloride equivalent with R (Col. 2) the same in all plants as in plant No. 1.

chloride. The data for all plants are averages from 1941 records. Table 4 indicates that the first plant (Item 1) is the only one that definitely saved ferric chloride and money by using ferric and lime. Aside from the emphasis placed on filtration economies at the former plant, it must be remembered that the low ferric demand of the sludge and the higher price ratio (R) combine to produce most of this result.

The use of lime and ferric chloride on the digested sludges in the remaining plants, with their higher alkalinity and lower cost ratios, is further complicated by both higher ratios of lime to ferric chloride necessary to meet the higher iquid demand and lower price ratios. Consequently the economy of this procedure becomes more difficult and somewhat questionable. The low lime to ferric chloride ratio (Col. 5, Table 4) for the Southerly plant at Cleveland (Item 4) is not quite typical. On a few occasions during 1941 it was necessary to increase the ferric dose to make up for the absence and scarcity of lime supply. In 1940 the average was 3.5 as compared with 2.8 in Col. 5, Table 4.

The values in Col. 6 are determined by Eq. 10*a*; those in Col. 10 are determined in the same manner when computing lime and ferric chloride percentages on dry cake weight. The values in Col. 7 are computed by the same equation when R is 4.1 in all cases. This places all plants on the same price ratio basis as Minneapolis-St. Paul and removes this particular advantage this plant has over the others in determining Col. 6 values.

Eq. 5 was used on the yearly average sludge assays for the values in Col. 11 and decimals were eliminated in the tabulation; for example, the Westerly and

Southerly plants were computed as 5.7% and 4.8% of ferric chloride, respectively. On this basis the average calculated doses show a rather close approximation to the average of Col. 7 and a closer agreement with the average values in Col. 10, Table 4.

In a way this should be expected. In most cases of Table 1, where ferric chloride alone was used in developing Eq. 5, the ferric chloride doses have been determined on dry cake solids. This chemical increases the weight of cake solids very little. In using lime and ferric chloride this is not the case. In this practice the cake solids are increased sufficiently to account for the differences between Cols. 6, 10, and 11, Table 4. Furthermore, in such installations using ferric chloride and lime, the actual sludge solids handled are estimated both from weighed cake and gallons of sludge sent to the filters. In some instances this procedure may result in an error. The error should be less when calculations are made on cake solids actually weighed and determined through accurate sampling and moisture determinations despite the fact that integrated weightometer readings are frequently light.

In this case, Cols. 10 and 11, Table 4, indicate that Eq. 5 is approximately valid for sludges dosed with lime and ferric chloride when the combined percentages of these chemicals on cake solids are converted to the equivalent percentage of ferric chloride according to Eq. 10*b*. Furthermore, at price ratios of 5 and 6, common to smaller plants using crystal ferric chloride, Eq. 10*a* will be found relatively accurate.

Therefore, if Eq. 5 or its nomograph, Fig. 4, is to be used as a guide for careful dosing of lime and ferric chloride with essential assays known, Eq. 10*b* also will be of value, for then the percentage addition of lime (L) will be:

$$L = R\,(C - F) \qquad (10c)$$

with C determined either by the nomograph or careful tests, and R from local prices.

Summary

Eq. 5 has been developed with the purpose of increasing current knowledge of the basic factors that influence coagulant requirements in sludge conditioning. Research indicates that these factors are broadly two in number: (*a*) The liquid demand of the chemically and biochemically contaminated water associated with the sludge solids; and (*b*) the demand of the wetted sludge solids themselves. In this paper, the former is based on the scientific weight relationship of bicarbonate alkalinity to ferric chloride, whereas the latter is based on an empirical relationship of ferric chloride to the volatile or organic portion of the sludge solids.

In determining both of these demands experimentally, the writer has purposely chosen only those plant records and experimental results that are pacemaking in their efforts to save conditioning chemical. Such a choice of statistical averages does not involve dosing sludges to produce the best filter yields and driest cakes in actual practice. With these facts in mind it is believed that the second part of Eq. 5 clearly indicates that the solid demand for ferric chloride and its chemical substitutes is a function of a constant times the ratio of

volatile matter to ash in the sludge solids. Undoubtedly the ratio of water to sludge solids in the first part of the equation is likewise determined by this same ratio. As the volatile-ash ratio only approximates the true ratio of organic to mineral matter the true ratio may disclose more meaning to the constant evolved. For general mathematical purposes the formula for total ferric chloride requirements (X) when using alkalinity assays is as expressed by Eq. 6, in which K is the only empirical constant. According to the degree to which the liquid and solid demand of a sludge is combined with ferric chloride prior to filtration, constant K probably varies from the value of the first constant (1.08) to some multiple of this value. For careful vacuum filter practice it is approximately 1.6 (see Eq. 5).

Such careful practice primarily depends on constancy of sludge characteristics and operating averages. To designers of vacuum filter installations either Eq. 5 or 6 should be of definite aid in choosing the best type of sludge processing to produce the lowest values for both liquid and solids demands and to gain constancy of operating averages. However, the operator's chief aid in taking economic advantage of the constancy of sludge characteristics through careful filter operation is the installation of coagulation and filtration equipment that does not tend to destroy the coagulated sludge floc and keep the conditioned sludge unduly long in process before reaching the filter cloth.

Acknowledgment

For data generously contributed, acknowledgment is due to: George J. Schroepfer, Assoc. M. Am. Soc. C. E., Minneapolis-St. Paul Sanitary District; Ralph E. Fuhrman, Assoc. M. Am. Soc. C. E., Washington, D. C.; Benjamin Benas, M. Am. Soc. C. E., San Francisco, and John D. McDonald, Springfield, superintendents in charge of their respective plants; George E. Symons, Assoc. M. Am. Soc. C. E., former chief chemist, Buffalo Sewer Authority; Paul D. McNamee, chief chemist, Washington, D. C., Sewage Treatment Plant; Robert J. Ross, M. Am. Soc. C. E., Manager of the Metropolitan District Commission, Hartford; and W. M. Scott, Chairman of Commissioners, Greater Winnipeg Sanitary District.

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DISCUSSIONS

ERECTION OF HYDRAULIC TURBINES AND GENERATORS

Discussion

BY ANDREW M. KOMORA

ANDREW M. KOMORA,[9] M. AM. SOC. C. E.[9a]—As Mr. Roberts states, the grouting of embedded parts of a turbine is highly important. It is regretted that, because of space limitations imposed by the restrictions of World War II, it was impossible to include the results of the experiments conducted at Wilson Dam with various grouting procedures. Perhaps these results can be published later as a special phase of turbine erection.

Construction men, especially concrete-placing crews, have a mania for using vibrators close to the turbine-embedded parts. For proper engineering use small vibrators can be permitted under turbine parts if they are securely anchored.

The suggestion made by Mr. Roberts that the turbine and shaft can be raised into position by means of hydraulic jacks is a good one. In certain cases, however, it is more feasible to use temporary coupling bolts as the design of the unit does not lend itself to the use of hydraulic jacks. As stated by Mr. Roberts, it cannot be overemphasized that large-diameter speed rings and pit liners must be secured and braced at close intervals to preclude the possibility of distortion due to concreting.

As emphasized by Messrs. Morgan and Brainard, the use of hinged side arms of a girder-type thrust bearing should eliminate the tendency to pinch the guide bearing when the unit is operating under load. The writer agrees with Mr. Brainard that the coupling bolts should be stretched only slightly in excess of the load imposed on the bolts by weight of the water-wheel parts, the hydraulic thrust, and the cylinder oil pressure, if any. In many installations, the stretch required to take such loads will produce a stress in the bolts close to the allowable working limit.

The only possibility that the turbine and generator shafts made by different

NOTE.—This paper by Andrew M. Komora was published in March, 1944, *Proceedings*. Discussion on this paper has appeared in *Proceedings* as follows: June, 1944, by Paul E. Gisiger, and Frank I. Morgan; and September, 1945, by J. F. Roberts, and D. E. Brainard.

[9] Asst. Engr. Director, Republic of Peru, Central Hidroeléctrica del Cañón del Pato Project, Lima, Peru.

[9a] Received by the Secretary November 4, 1944.

manufacturers will not match is the one chance that the coupling faces might not be at right angles to the axis of the shaft. The writer has yet to see an instance in which they failed to be at right angles. It may be accepted that the cost of reaming the coupling holes in the field is far less than that of shipping the entire shaft from one plant to another.

The writer agrees with Mr. Brainard that the rotation check has little, if any, significance in the case of integral thrust collars. It is still of great value in installations where the thrust collar is an independent unit and is either keyed or shrunk on to the shaft.

Thrust collars, forged integral with the shaft, have been used on units up to 60,000 kw with very good results. In general, the designs that are used on integral thrust collars present fewer erection problems to the field forces.

The writer wishes to acknowledge, gratefully, the discussions of Messrs. Gisiger, Morgan, Roberts, and Brainard.

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DISCUSSIONS

CONSTRUCTION OF THE FLOW NET FOR HYDRAULIC DESIGN

Discussion

BY CLARENCE S. JARVIS

CLARENCE S. JARVIS,[5] M. AM. SOC. C. E.[5a]—This presentation of the flow-net problem by Mr. Foster is original in approach and concept. Too often, however, a tendency toward abstract expression is confusing, and a more elementary presentation might prove acceptable to nonspecialists. In the structural feature employed by the author, for example, it seems preferable to concentrate on: (*a*) Access capacity over the "morning-glory" spillway rim; (*b*) outlet capacity of the stem or throat, leading to the open drainage channel below the dam; and (*c*) important phenomena associated with normal operation.

The usual behavior of such structures—laboratory models and also ordinary funnels—indicates that considerable turbulence and entrapment of air will occur where the converging filaments of swiftly flowing water crowd the outlet stem. Perhaps hydrostatic head will be built up as required to dispose of the changing overflow at the upper rim. Pressures resulting from the turbulence and the entrapped air doubtless would exceed the total pressures subjected to variations due to changing velocities of flow over concrete or some other structural surface.

Surely the first investigations should concern the discharge capacity over the rim, in accordance with the author's presentation. Thus:

$$Q = 3.28\, L\, H^{3/2} \qquad (28a)$$

or for a full circle,

$$Q = 10.36\, D\, H^{3/2} \qquad (28b)$$

For the example under consideration, in which the diameter is 68.9 ft, Eq. 28*b* yields

$$Q = 714 \times H^{3/2} \qquad (28c)$$

NOTE.—This paper by H. Alden Foster was published in May, 1944, *Proceedings*. Discussion on this paper has appeared in Proceedings, as follows: January, 1945, by J. W. Howe.

[5] Senior Civ. Engr., Intelligence Dept., Office, Chf. of Engrs., Washington, D. C.

[5a] Received by the Secretary January 27, 1945.

For $H = 9$ ft in Eq. 28*c*, Q becomes 19,300 cu ft per sec; or, for $H = 16$ ft, $Q = 45{,}700$ cu ft per sec.

Until the engineer can compute the vibration due to the inevitable shock of the readjustment of the section by the descending sheet of water and the motion for steady discharge in a nearly horizontal conduit before the discharge escapes into an open channel, it seems unimportant to determine the slight changes in velocity and related pressures, while totally neglecting friction, impact, and eddy losses. According to the author's statement in the concluding sentence of the "Summary," "* * * the flow-net method is not applicable in cases where friction and impact or eddy losses may be expected to have a controlling effect on velocities and pressures." Surely friction, impact, and eddy losses are not negligible, or the author's estimate for the discharge capacity of spillway lip under a head of about 8.90 ft could not be taken as 21,500 cu ft per sec—more than 10% in excess of the quantity derived (8*a*)[5b] from the use of Eq. 28*a*.

Considerations of economy would probably so limit the dimensions of the spillway stem and horizontal outlet conduit that it would be questionable whether a flow of 45,500 cu ft per sec could be accommodated. If the conduit capacity were somewhat less, it would become the controlling factor, causing a relatively quiet pondage over the entire morning-glory area except near the center, where the tendency to circular motion might establish the well-defined vortex so familiar to observers of eddies in turbulent stream surfaces. Obviously, the neglected factors of friction, impact, and eddy losses would control to such an extent that the length as well as the cross section of the outlet conduit would have to be considered in the determination of the actual capacity of the spillway under a given head of water in the reservoir and also under the total head above the downstream end of the outlet conduit.

Such theoretical studies and analyses should be encouraged, but the results should be reduced to a practical working basis.

Bibliography.—

(8) **"Tests of Circular Weirs"** by Cecil S. Camp and J. W. Howe, *Civil Engineering*, April, 1939, p. 247. (*a*) Eq. 3, p. 248.

[5b] Numerals in parentheses, thus; (8*a*), refer to corresponding items in the Bibliography (see Appendix I of the paper), and at the end of discussion in this issue.

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DISCUSSIONS

TRANSPORTATION OF SUSPENDED SEDIMENT BY WATER

Discussion

BY A. A. KALINSKE

A. A. KALINSKE,[54] ASSOC. M. AM. SOC. C. E.[54a]—Data from an extensive series of measurements of the transportation of suspended material by water in an open channel are presented in this paper. The writer does not in any way question the value or accuracy of these data; however, he does seriously disagree with many of the conclusions drawn from the data, especially with the conclusions in regard to the effect of suspended sediment on the turbulence and hydraulic characteristics of the water flow. Specifically, the paper contains no real concrete evidence to prove the validity of the conclusions relating to: (1) The action of random velocity fluctuations on suspended sediment; (2) the effect of suspended sediment on the von Kármán k-values; and (3) the hypothesis that suspended load reduces resistance to flow.

So far as similarity of the transfer of sediment by turbulence to the transfer of momentum in an open water channel is concerned, the writer does not believe any too definite statements can be made as yet—for the one important reason that it is difficult to calculate the correct value of the momentum-exchange coefficient. All such computations must be based on some assumption as to the value and distribution of the shear. It is doubtful if, in a channel such as that used by the author, the shear is distributed linearly, due to the side and free surface effects and secondary currents. The author calculated the value of ϵ_m using the logarithmic velocity distribution law, which is not quite correct either near the water surface or near the bottom. Thus, the value of ϵ_m in the paper may not necessarily be the true value of the coefficient of momentum transfer. Any rigid comparison between ϵ_m and ϵ_s, therefore, is not really warranted.

The author's remarks regarding the presence or absence of correlation be-

NOTE.—This paper by Vito A. Vanoni was published in June, 1944, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: December, 1944, by Ralph W. Powell, and E. R. Van Driest; and February, 1945, by Weston Gavett, and Berard J. Witzig.

[54] Associate Prof., Univ. of Iowa, and Associate Director, Iowa Inst. for Hydr. Research, Iowa City, Iowa.

[54a] Received by the Secretary February 3, 1945.

tween the turbulent velocity components are somewhat confusing. The correlation that exists when momentum is exchanged in two-dimensional flow is the relationship between the vertical component v' and the horizontal component u'. The presence or absence of any such correlation has no relation to the transfer of sediment since the horizontal component u' has nothing to do with keeping sediment in suspension. If the shear is zero, the correlation between u' and v' is zero, since, according to Reynolds, the shear in the turbulent flow can be expressed as

$$\tau = \rho \overline{u' v'} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (33)$$

However, this does not mean that the turbulent exchange coefficient is zero. Thus, in the center of a pipe the shear is zero, but ϵ has a finite value since heat or mass can be transferred across the center of a pipe even though momentum cannot be transferred. So far as the writer knows, the only way to obtain an accurate idea of the transfer coefficient in a region of flow where the shear is difficult to estimate is by direct measurement using diffusion experiments. The writer has reported such measurements elsewhere.[35]

The writer's chief criticism of the author's laboratory procedure is that the experiments were not made with the channel bed completely covered with the sand being transported in suspension. If the bed is not completely covered with unconsolidated sand, a particular flow is not transporting all the sediment in suspension that it is capable of transporting. Of course, there is always some loose sand on the bottom; however, the patches of sand are constantly changing and, of course, are influencing the bed roughness. Thus, the bed roughness for one discharge may be different from that for another discharge. In other words, the bed undoubtedly differed for the various runs, thus very likely producing the effects on velocity, friction factor, and k that the author attributes to changes in concentration or size of material in suspension.

For the study of the transportation of sediment in suspension and of the effects that this suspended sediment may have on the hydraulic characteristics of the water flow, it is necessary that the flow take place over a bed entirely covered with the material being transported in suspension and that the suspended matter be in equilibrium with such a bed. Unless this is done, the data obtained will be useful only for the study and analysis of the vertical distribution of suspended sediment in turbulent flow.

In similar experimental studies made by the writer, the channel bottom was always covered with the material being moved in suspension and the results from these studies do not in any way confirm the author's conclusions. In 1943 an extensive series of experiments was made at the Iowa Hydraulics Laboratory, University of Iowa, Iowa City, on the transportation of a fine sediment (median diameter 0.011 mm) in a flume $2\frac{1}{4}$ ft wide with about $\frac{1}{2}$ ft of water.[55] Runs were made with water velocities varying from 0.84 to 2.73 ft per sec and with sediment concentrations varying from 0.64% to 11.1% by weight. Some

[35] "Experiments on Eddy-Diffusion and Suspended-Material Transportation in Open Channels," by A. A. Kalinske and C. L. Pien, *Transactions*, Am. Geophysical Union, Pt. II, 1943, pp. 530–536.

[55] "Study of Transportation of Fine Sediments by Flowing Water," by A. A. Kalinske and C. H. Hsia, *Bulletin No. 29*, Univ. of Iowa, Iowa City, 1945 (in press).

of the conclusions drawn from these studies were as follows:

(1) The velocity distribution in the center of the channel followed the logarithmic law very well and no difference was noted even when the sediment concentration was 11.1%;

(2) Manning's n was influenced by the form of the bed, which varied with the bottom shear, and not by sediment concentration; and

(3) The value of the von Kármán k-factor changed from 0.32 to 0.44 without any regular variation with respect to sediment concentration.

The sediment concentrations in the studies at the Iowa Hydraulics Laboratory were considerably higher than those in the author's experiments and also had a wider range. However, the size of the material used was very much smaller. The writer is forced to state that the results of the studies at the Iowa Hydraulics Laboratory do not seem in any way to confirm the author's conclusions which apparently were intended to be quite general in scope.

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DISCUSSIONS

HYDRAULIC MODEL INVESTIGATION OF MITER GATE OPERATION

Discussion

BY WILLIAM A. TRIPP

WILLIAM A. TRIPP,[6] ESQ.[6a]—The model tests described by Mr. Amster do not include two factors, present in the actual operation of the locks of the Panama Canal, which have a substantial effect on the operating loads. These are: (1) The influence of the 200-ft chamber on the effective length, or free surface, of the chambers; and (2) the use of the culvert in equalizing water levels. Without proper evaluation of these factors, it is felt that the tests do not establish a sound criterion for predicting the forces on the prototype and are of doubtful value for application to other installations.

An analysis of the effect of these factors made by the writer indicates the factors to be much more important than has been recognized. According to this analysis, failure to introduce the 200-ft chamber properly into the problem appears to produce values of torque that are approximately 100% too large, and failure to include the effect of the culvert appears to produce values that are 50% too large. The combination of these two factors indicates that the tests may vary by as much as 200% from a correct prediction of the normal operating torques of the prototype. The method of analysis used by the writer is believed to be sound as it has been checked against sample cases taken from both the model and the prototype tests on the Panama Canal, yielding results which are reasonably close.

The basic conditions selected for the model tests include an upper chamber of 1,105 ft and a lower chamber of 200 ft. The 200-ft chamber is only an intermediate chamber between two 1,105-ft chambers; and the gates at both the upper and lower ends of this short chamber are operated simultaneously. This method of gate operation makes the short chamber act hydraulically much like a pool with an infinite free surface, and materially reduces the hydraulic

NOTE.—This paper by Maurice N. Amster was published in March, 1944, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: June, 1944, by Edward Soucek; and September, 1944, by Isaac DeYoung, and F. W. Edwards.

[6] Elec. Engr., Chas. T. Main, Inc., Boston, Mass.; formerly Senior Engr., Mech.-Elec. Section, Special Eng. Div., The Panama Canal, Diablo Heights, Canal Zone.

[6a] Received by the Secretary January 27, 1945.

resistance offered to the operation of each gate, at least during the opening cycle—the maximum duty cycle.

The free surface of the lock chambers has an important influence on the forces produced in the gate machinery. Evidence in support of this fact is presented in Fig. 8, in which the test curves for the opening cycle show progressively higher torques with shorter chamber lengths. Mr. Amster discusses this condition briefly under the heading, "Test Results." He states that increasing the length of the larger chamber beyond some 625 ft did not have a material effect on the torque. It should be noted, however, that the 200-ft chamber by then would be the dominating influence in the model. The torque required to move a gate is caused largely by the differential head produced by the movement of the gate. In effect, the gate is a wall between the two chambers on each side of it. Movement of the gate is equivalent to a change in the sizes of the two chambers, and, starting with a given quantity of water in each chamber, a differential head will be set up until sufficient water can flow to equalize the water levels. With a gate of a given size, a given movement will represent a definite volumetric change, which must be distributed over the free surface of the lock chambers, raising the water level in the chamber above the gate and lowering it in the chamber below the gate. The larger the free surface of each chamber, the smaller will be the differential head produced by the quantity of water involved. Thus, the chamber area will have a direct influence on the machinery torques for a given gate and gate drive. By the same reasoning, a gate at the other end of one of the chambers which follows the movement of the first gate, and causes a displacement of water compensating for the displacement of water caused by the first gate, will substantially prevent any change in the water level in the intermediate chamber. This is what takes place in the 200-ft chambers in the prototype at the Panama Canal.

Of course, the method of operation assumed for the model could be encountered in the prototype if something happened to prevent the "following" gate from operating, or if a gate were operated in this manner while being serviced. However, failure of the "following" gate to operate would be noted readily by the operator and, in any event, would impose only a short overload on the drive machinery. Such a failure would slow down the gate operation, thereby partly relieving the condition. Servicing operations could, and ordinarily would, be taken care of by nonsynchronous operation of the gate leaves. A substantial reduction in design requirements can thus be effected, by basing the size of the power unit on normal operation, and making the drive mechanism physically strong enough to take any momentary overloads that might be encountered with this smaller power unit.

With regard to the culvert, the operating practice at the Panama Canal is to keep the culvert valves open while swinging a set of gates. The importance of this procedure is evident from the discussion of the differential head produced by the movement of the gate. The significant fact is that the head is relieved by an equalizing flow of water between the chambers. Three channels are available for this flow—(*a*) the opening between the gate leaves, (*b*) the opening

over the sill, and (*c*) the culvert. The first two start at zero for an opening cycle, whereas the third is constant. The importance of the culvert will depend upon its dimensions in relation to other geometrical features for the particular case. In the existing prototype at the Panama Canal, the culvert area is greater than the combined area of the other two channels for about the first 5% of the arc of gate swing in opening. It is understood that these relationships are about the same in the new locks. It is particularly significant, therefore, that the tests indicate that the peak of torque occurs within about the first 8% of the arc of swing. Of course, the rate of water flow per unit of cross-sectional area will be less for the culvert than for the gate opening, but the relative magnitude represents an important factor during a critical part of the cycle of operation. As is generally well known (and the model tests prove this), with lock gates following the kinematics of the Panama Canal type and with well-designed wall recesses, the torques determining the design of the gate machinery occur during this early part of the opening cycle. Anything that can be introduced to decrease torques during this period will have a substantial influence in reducing design requirements. Culverts of the relative size of those at the Panama Canal are of sufficient importance to warrant full and careful evaluation.

The magnitudes of the influences of the two factors discussed herein will vary for different physical relationships and operating conditions. They appear to be about the same for both the old and the new locks at the Panama Canal. Their relative importance indicates that these magnitudes should be introduced into any analysis or tests involving similar arrangements.

The writer wishes to acknowledge and express his appreciation of the interest and assistance of B. O. McCoy, Assoc. M. Am. Soc. C. E., in the study and analysis of this problem.

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DISCUSSIONS

LOCK MANIFOLD EXPERIMENTS

Discussion

BY J. PIETRKOWSKI

J. PIETRKOWSKI,[11] ESQ.[11a]—Elsewhere[12] the writer has developed equations for open channel flow, with $k_d = 1$, which were based on the same logic as Eq. 1. The water surface was found to rise in downstream direction corresponding to the pressure increase in the closed conduit treated by the authors.

Experience has shown that the water surface in the channel in the region of the side spillway is not horizontal but that it slopes toward the outlet. The effect of this phenomenon cannot be computed theoretically but must be appraised by the experimental determination of coefficients such as C_d in the conduit treated by the authors. Both the investigation reported by the authors for closed conduits and that reported by the writer for open channels can be interpreted best by instruction sheets, outlining steps in a computation, rather than by formulas.

Two differences between open channel flow and conduit flow should be emphasized:

(1) To be of any value, the spillway crest must be much longer than the corresponding dimension of the port; therefore the friction loss in the channel along the weir must be considered.

(2) The critical depth $d_c = \frac{V^2}{g}$ may occur in the channel, adjacent to the side spillway, with important consequent results upon the discharge over the weir.

It is noteworthy that the authors report the pressure change with $k_d = 1$, would be twice the difference between velocity heads upstream and downstream from the port. They conclude that Bernoulli's theorem is not applicable to problems of this kind. Similarly, the writer could show that the slope of the energy line in an open channel, computed by the momentum equation, is twice

NOTE.—This paper by Edward Soucek and E. W. Zelnick was published in October, 1944, *Proceedings*.

[11] Jerusalem, Palestine.

[11a] Received by the Secretary February 9, 1945.

[12] "Lateral Overflow Weirs," by J. Pietrkowski, *Journal*, Assn. of Engrs. and Archts. in Palestine, October 1942, p. 7.

as great as the slope computed by the energy equation. Again Bernoulli's theorem is not applicable.

Incidentally, Bernoulli was the last man to expect too much from his famous theorem; he was the first to understand that conditions at the outlet of a vessel, or conduit, can be described adequately only by considering momentum.[13] L. Föppl has contributed a valuable statement[14] regarding Bernoulli's theorem —namely, that this theorem is valid either along a streamline even if curl occurs or between different streamlines when the curl is zero. The lock manifold and the side spillway are surely phenomena involving different streamlines with curl.

[13] "Hydraulik," by Philipp Forchheimer, B. G. Teubner, Leipzig and Berlin, 1914, Eq. 187b, p. 337.
[14] "Aufgaben aus Technischer Mechanik," by L. Föppl, 6th Ed., Vol. IV, Munich and Oldenburg, 1930, p. 385.

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DISCUSSIONS

EVAPORATION FROM A FREE WATER SURFACE

Discussion

BY C. W. THORNTHWAITE

C. W. THORNTHWAITE,[5] ESQ.[5a]—Although the author is reporting original observations of evaporation, the most valuable contribution of his paper is probably in showing the possibility of developing an evaporation equation in terms of mass transfer of moisture through a layer of the atmosphere. For more than two decades meteorologists have been investigating turbulence in the lower atmosphere for the purpose of determining the mechanism of mass transfer of momentum, heat, moisture, and dust. The literature on the subject is already very extensive; but few engineers have given much attention to this development.

The author compares the transfer of moisture from an evaporating water surface to the process of heat transfer from a solid to a gas. He states (see heading, "How Evaporation Occurs"):

> "* * * A boundary layer of stagnant gas surrounds the solid through which heat is transferred by conduction alone. Outside the boundary layer, the heat is transferred by convection, frequently aided by turbulence. Heat transfer has been computed successfully by the use of an over-all transfer coefficient, combining the coefficients of conduction and convection, known as a film coefficient."

The temperature difference used is that between the wall temperature of the solid boundary and the temperature of the gas at a considerable distance. The author believes that the rate of transfer of water vapor through the atmosphere could be calculated in a similar manner. Whether or not it has been possible to combine the coefficients of conduction and convection in the computation of heat transfer need not be considered at this time. It is important to understand, however, that the successful determination of the flow of heat through a layer would not mean that moisture transfer could be computed similarly.

NOTE.—This paper by G. H. Hickox was published in October, 1944, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: February, 1945, by Carl Rohwer.

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[5a] Received by the Secretary February 16, 1945.

John Leighly (45)[55b] has outlined clearly the difficulty of computing evaporation in this way, and the discussion immediately following is adapted from his paper. Conditions for the transfer of matter or energy within the boundary layer are totally different from those which prevail in the zone of turbulence above that layer (see Fig. 14). When air is turbulent, its properties are trans-

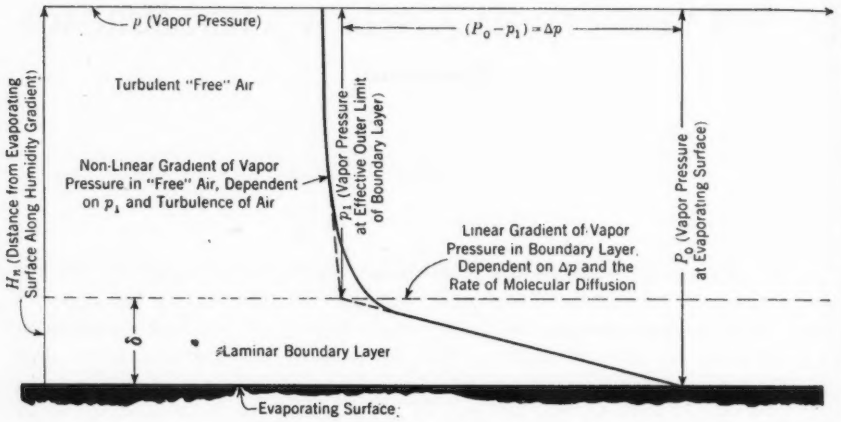
p (Vapor Pressure)
$(P_0 - p_1) = \Delta p$
Turbulent "Free" Air
H_n (Distance from Evaporating Surface Along Humidity Gradient)
Non-Linear Gradient of Vapor Pressure in "Free" Air, Dependent on p_1 and Turbulence of Air
p_1 (Vapor Pressure at Effective Outer Limit of Boundary Layer)
P_0 (Vapor Pressure at Evaporating Surface)
Linear Gradient of Vapor Pressure in Boundary Layer, Dependent on Δp and the Rate of Molecular Diffusion
δ
Laminar Boundary Layer
Evaporating Surface

FIG. 14.—CHARACTERISTIC DISTRIBUTION OF HUMIDITY IN THE LAMINAR BOUNDARY LAYER NEXT TO AN EVAPORATING SURFACE AND IN THE ADJACENT PARTS OF THE OVERLYING TURBULENT LAYER

ported rapidly between layers by the actual exchange of bodies or masses of air, and except under special conditions steep gradients of velocity, temperature, or admixtures cannot exist. In the laminar boundary layer, however, where no mixing by movement normal to the boundary surface takes place, heat may be transferred only by molecular conduction; momentum may be transferred only by molecular friction; and gaseous admixtures, such as water vapor, may be transferred only by molecular diffusion. Therefore, gradients are characteristically steeper in the boundary layer than in turbulent air. Rates of molecular transfer—coefficients of heat conduction, molecular friction, and diffusion—are nearly constant within the thickness of the boundary layer, and this constancy of rate of transfer produces sensibly linear gradients of temperature, velocity, and humidity from the bounding surface out to the limit of the boundary layer. Outside the boundary layer, in the region of turbulent mixing, molecular transfer is negligible in comparison with molar transfer through turbulence. The intensity of turbulent mixing varies greatly with distance from the bounding surface. Hence, gradients are variable in the turbulent parts of the air, normally decreasing with increasing distance from a bounding surface, as the intensity of turbulent interchange increases.

When considering the proportionality of rate of evaporation to the differ-

[55b] Numerals in parentheses, thus: (45), refer to corresponding items in the Bibliography (see Appendix I of the paper), and at the end of discussion in this issue.

ence in vapor concentration at the water surface and at a distance, Mr. Leighly showed that the simplest expression of such proportionality is

$$E = C\,(P_o - p_1) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (25)$$

in which E denotes mass of water lost (or gained) by unit area in unit time; C is a proportionality coefficient; P denotes vapor pressure at saturation; p denotes actual vapor pressure; and the subscripts o and 1 denote location at an evaporating surface and in the air, respectively. The equation would be complete if humidity difference were the only variable. This simple equation for E has the familiar form of many equations that express rate of transfer of matter or energy through space—conduction of heat, diffusion of solutes, etc.—along a gradient; but the difference $(P_o - p_1)$ can define a gradient only when it is divided by a term defining the distance between the points of observation of moisture concentration. If this distance is designated z, and the change in moisture concentration along it is linear, the gradient is defined by $(P_o - p_1)/z$. Therefore, the vapor-pressure difference alone can be used as the basis of an evaporation formula only if z is constant for all evaporating surfaces. Since z cannot be constant, it must be included in any evaporation formula that uses the difference $(P_o - p_1)$. Even then, the expression $(P_o - p_1)/z$ accurately describes the gradient only if change in vapor pressure normal to the evaporating surface is linear. Within the turbulent layer and thus for unlimited distances from the evaporating surface, the gradient is not constant. The only satisfactory general expression of the gradient is therefore dp/dz, which is the change in moisture concentration with respect to height.

To determine the rate of flow of heat through unit cross section of a window pane, not only must one know the coefficient of thermal conductivity of the glass and the difference in temperature between the inner and outer surfaces, but also the thickness of the glass. This is easily understood; yet from the time of John Dalton, evaporation formulas have given the rate of flow of water vapor in terms of a difference in vapor concentration at the water surface and at an undetermined distance.

One of the finest investigations of evaporation ever conducted in the United States was that by Carl Rohwer, M. Am. Soc. C. E., reported in 1931 (16). All observations used in the derivation of his formula were carefully made under rigorously controlled conditions. He used an aspiration psychrometer to determine the vapor pressure of the air within an inch of the water surface. His laboratory experiments were conducted in still air and with controlled wind. The general form of his evaporation formula is given in Eq. 25. He found that the coefficient C increased linearly with increasing wind velocity according to the relation,

$$C = (0.44 + 0.118\,V) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (26)$$

in which V = wind velocity, in miles per hour. Actually the coefficient C contains both the coefficient of moisture transfer and the distance term that must be associated with difference of moisture concentration in order to define the moisture gradient. Mr. Rohwer's measurements of moisture concentration

in the air, although made within an inch of the water surface, were well within the turbulent layer; but, since they were made under controlled conditions, the fact that two layers with very different transfer coefficients were involved does not vitiate the results. With increased wind velocity the coefficient of turbulent exchange becomes larger and the laminar layer decreases in thickness. Thus both factors work together to increase the value of the coefficient of proportionality C.

With outdoor observations of evaporation made under uncontrolled conditions, Mr. Rohwer found his formula to be less satisfactory. He recognized that both wind velocity and atmospheric moisture are highly variable in the free air and he rightly attributed some of the inconsistencies in the data to this variability.

The author presented a general formula (Eq. 2*b*) for mass transfer of water vapor which uses a gradient of vapor concentration rather than a difference. Nevertheless, there is no indication that he measured a gradient in the course of his experiments. He refers to vapor concentration "at a distance" (see heading, "Evaporation as a Mass Transfer Process: Experimental Data"). He measured the relative humidity of the air with a standard sling psychrometer. When using this instrument, the height of the observation cannot be specified nor the moisture gradient determined with any degree of accuracy. Furthermore, it is not possible to use a sling psychrometer without disturbing still air and profoundly modifying its moisture structure. Apparently all other measurements were made with great care. Considering the crudeness of the one, such accuracy in the others was scarcely justified.

The author has stated in the paragraphs following Eq. 15 that the writer and his associates do not feel that it is possible to develop a satisfactory equation for evaporation in terms of the "difference in vapor concentration at the water surface and at a distance." As a matter of fact, the evaporation equation introduced by the writer and his associates does employ a difference (46*a*). Both observations of moisture concentration were made within the turbulent layer, and the distance element of the moisture gradient was incorporated into a coefficient of proportionality.

The author greatly underestimates the difficulty of the problem when he states (see heading, "How Evaporation Occurs") that the rate of moisture transport can be determined by use of an over-all transfer coefficient, combining the coefficients of conduction and convection. In the first place, the coefficient of turbulent exchange increases with height in the free air and varies not only with wind velocity but also with its stability structure. In the second place, the thickness of the laminar layer, in which the coefficient of molecular diffusion applies, becomes less as the intensity of turbulence in the turbulent layer above it increases.

The most difficult problem is to obtain the turbulent transfer coefficient in terms of some measurable quantity such as wind velocity or temperature. This has been accomplished for atmospheric conditions when the wind velocity varies as the logarithm of height (46*b*). The writer and his associates (47)(48)(49), however, have presented measurements of wind velocity at different heights to show that the logarithmic law of wind distribution is of restricted validity.

The data show that the law applies very well when the temperature lapse rate is adiabatic (or, within a shallow layer, isothermal), but that it fails when the lapse rate is either superadiabatic or stable. When plotted on a semilogarithmic scale, only the late morning and early evening observations fall along a straight line. The daytime observations plot along lines having concave curvature; those made at night show convex curvature.

The writer concluded that when the thermal lapse rate is adiabatic, as for brief periods in the morning and again in the evening, the logarithmic law is fulfilled and the coefficient of turbulent mixing derived from it is valid. At this time turbulent mixing is mechanical only.

In the middle of the day, when the ground becomes heated and superadiabatic lapse rates may become very large and thermal convection is actively induced, convective turbulence is superimposed upon mechanical turbulence and the turbulent exchange coefficient becomes greater, perhaps much greater, than the logarithmic law would indicate. The concave curvature of the wind gradients is a consequence of this increase in the coefficient of turbulent mixing. As momentum is transported downward more actively, the tendency for differences in velocity in adjacent layers to be equalized increases, and the wind profile becomes more nearly vertical except very close to the ground surface.

At night when the ground, having been cooled by radiation, cools the adjacent air, the lowest layer of air becomes cooler than that above it and a stable lapse rate is created. The air next to the ground, being cooler, is denser than that above and, if moved mechanically upward into a faster moving layer, would tend to move down again instead of mixing with the air in the new environment. In this instance the coefficient of turbulent transfer is less than that specified by the logarithmic law. Momentum is transported downward less actively; the tendency for velocity differences in adjacent layers to be equalized diminishes; and the velocity profile departs more than ever from the vertical except within a foot or two of the ground.

The fact that the logarithmic law of wind structure is valid only when an adiabatic temperature structure prevails means that the problem of determining the coefficient of turbulent transfer is much more complicated than had been anticipated. First, the wind velocity gradients themselves must be determined more accurately than is possible with ordinary anemometers. In addition, it will probably be necessary to make measurements of temperature at various heights in order to establish the degree of stability of the atmospheric layer through which the water vapor is moving.

The writer has not seen the literature referred to by the author in his statement (see heading, "How Evaporation Occurs"), "Heat transfer has been computed successfully by the use of an over-all transfer coefficient, combining the coefficients of conduction and convection * * *." He does not share the author's view, however, that "There seems to be no reason why the rate of transfer of vapor particles cannot be calculated in an entirely similar fashion." If it is possible to determine an over-all coefficient of heat transfer through both laminar and turbulent layers, it is only because the temperature structure directly controls the intensity of turbulent transfer and indirectly the thickness of the laminar layer. The temperature difference term of the equation of heat

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transfer, therefore, may contain a part of the transfer coefficient. Atmospheric moisture concentration does not share with heat the property of controlling the magnitude of the coefficient of turbulent transfer, however. Consequently, it appears to the writer that it will not be possible to integrate across the boundary layer. In computing the rate of transfer of water vapor from a natural surface there seem to be only two possibilities that are scientifically defensible—either determine the moisture gradient wholly within the laminar layer where the coefficient of diffusion applies, or wholly within the turbulent layer, where it will be necessary to utilize a coefficient of turbulent transfer (50*a*).

Bibliography.—

(16) **"Evaporation from Free Water Surfaces,"** by Carl Rohwer, *Bulletin No. 271*, U.S.D.A., December, 1931. (*a*) p. 9. (*b*) pp. 34–42. (*c*) p. 71.

(45) **"A Note on Evaporation"** by John Leighly, *Ecology*, Vol. 18, No. 2, April, 1937, pp. 180–198.

(46) **"Measurement of Evaporation from Land and Water Surfaces,"** by C.W. Thornthwaite and Benjamin Holzman, *Technical Bulletin No. 817*, U.S.D.A., May, 1942. (*a*) p. 22. (*b*) p. 14.

(47) *Transactions*, Am. Soc. C. E., Vol. 109 (1944), p. 669.

(48) **"Note on the Variation of Wind with Height in the Layer Near the Ground,"** by C. W. Thornthwaite and Maurice Halstead, *Transactions*, Am. Geophysical Union, Section of Meteorology, 1942, pp. 249–255.

(49) **"Wind-Gradient Observations,"** by C. W. Thornthwaite and Paul Kaser, *ibid.*, 1943, pp. 166–182.

(50) **"Atmospheric Moisture in Relation to Ecological Problems,"** by C .W. Thornthwaite, *Ecology*, Vol. 21, No. 1, January, 1940, pp. 17–28. (*a*) p. 23.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

COLUMN FORMULAS

Discussion

BY EDWARD GODFREY

EDWARD GODFREY,[12] M. AM. SOC. C. E.[12a]—The subject of this paper is of great importance to designers. With the exception of the Euler formula, rational column formulas are conspicuously absent from engineering works. The Euler formula is the one rational formula in common use, but it is applicable to slender columns only and has no meaning when applied to columns with a slenderness ratio usually specified as proper in ordinary structural work. Furthermore, column formulas in common use give totally false unit values when used for columns with slenderness ratios to which the Euler formula is applicable.

In spite of statements in column-design literature, the Euler load, instead of being the load a column can sustain in equilibrium, is the absolute maximum load—the load at which the column is certain to fail. The proof of this statement is quite simple. Assume that a column is in the shape of a bow with a finite versed sine or offset. The Euler load, by its derivation, will double that offset. It will not stop there, however. The column is still a bow with an offset and that offset will be increased again for the same reason that the first increase occurred. This process will continue indefinitely, or until the column has failed completely, since the increments are not decreasing. If the load adds one half to the original bow, the final bow will be only twice the original bow or offset. When $x = \frac{1}{2}$, the infinite series $1 + x + x^2 + x^3 + \cdots$, etc., is 2.

In the "Summary," Mr. Osgood disparages the use of one formula for short columns and another for slender columns. The writer does not agree with this opinion since it is on this very point that he has always differed with standard theories on the subject.

The criterion for shorter columns—those in structures such as bridges and buildings where the ratio of slenderness is properly restricted—is stress per square inch on the extreme fibers of the section. Based on the strength of the material of the column, the stress must not exceed certain values which are

NOTE.—This paper by William R. Osgood was published in December, 1944, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: February, 1945, by L. E. Grinter, George Winter, and Jonathan Jones.

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[12a] Received by the Secretary February 19, 1945.

limited by a factor of safety. The criterion for slender columns—those outside the limit of the slenderness ratio permitted by specifications—is not in any sense the compressive strength of the material of the column, since only the modulus of elasticity (a property nearly constant for the various grades of steel) is needed for the derivation of the formula.

There are two distinct phases to be considered in determining the strength of a compression member. Failure to appreciate this fact has caused much misapprehension. Formulas have been derived, and accepted, that attempt to combine the two phases of the strength of columns in utter disregard of the distinction between the two.

Every column of elastic material is a spring or bow as well as a shaft subject to endwise compression, and the strength of a column considered as a spring is independent of the ultimate or elastic strength of the material. In addition, the formula showing this strength is quite distinct from any formula based on ultimate strength of the material. The Gordon-Rankine formula has limited application. This formula is fraught with danger, if it is used in the range of slender columns.

That the straight-line formula for the strength of a column can be substantiated, rationally and theoretically, has been demonstrated elsewhere by the writer.[13]

In former years at least, European engineers used the Euler formula with a factor of safety of five. There is no reason for so high a factor of safety in the region where the Euler formula is applicable. The Euler formula is never applicable to short columns. A factor of safety of two, or even less, is ample for use in the Euler load, if the Euler formula is applicable, because the condition that causes failure is not reached until the load approaches the Euler value. Also, columns not in the slender bracket are likely to fail at the first observable sidewise deflection.

In tests described by C. P. Buchanan,[14] the deflection due to the load did not exceed 0.2 in. within the elastic value of the column for any column tested, and at the ultimate load, in many cases, this deflection did not increase.

In his previously cited work,[13] the writer shows the locus of an exceedingly complex formula and comparison with the Euler load in the slender range (with a factor of safety of two). The complex formula referred to is completely rational. It is based on the simple assumption that a column is bowed with an offset 1/300 of its length. The compressive load increases the bow by an infinite series of decreasing terms. The bow is not "frozen," as in the derivation of the Gordon-Rankine formula.

The formula is too cumbersome to use in practice, but the locus, up to 90 radii, is a straight line, and beyond this it deviates but little from a straight line in the range of structurally permitted columns. In the "slender" range it almost coincides with the Euler value with a factor of safety of two, which is ample for slender columns.

However, instead of being a reason for the use of one formula for both slender and short columns, it is a sound reason for the use of the ordinary straight-line formula for bridge and building columns and the Euler load for light struts, such as are used in transmission and radio towers.

[13] "Steel Designing" (Book Three of a Series on Structural Engineering), by Edward Godfrey, 1913, Chapter X, pp. 152–162.

[14] "A Series of Failure Tests of Full-Size Compression Members, Made for the Pennsylvania Lines West of Pittsburg," by C. P. Buchanan, *Engineering News*, December 26, 1907, p. 685.

AMERICAN SOCIETY OF CIVIL ENGINEERS

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DISCUSSIONS

UTILIZATION OF GROUND-WATER STORAGE IN STREAM SYSTEM DEVELOPMENT

Discussion

BY RAYMOND A. HILL

RAYMOND A. HILL,[3] M. AM. SOC. C. E.[3a]—It might be inferred from the author's presentation of the subject that the safe yield of any ground-water unit is a definite quantity, as it is in the case of any normal surface reservoir formed by construction of a dam across a flowing stream. The safe yield of such a reservoir can be defined as the maximum rate at which water can be withdrawn continuously between times of filling, because all of the inflow is captured, until the reservoir fills and overflows, and losses are restricted to evaporation from the water surface and nominal leakage.

In the case of a subsurface reservoir, however, the proportion of the contributing supply which is captured and the magnitude of losses through evaporation and leakage are dependent in large degree upon the depth at which the ground water is held below the surface; this depth in turn is dependent upon the draft which is made on this reservoir. The safe yield—that is, the rate at which water may be withdrawn from any ground-water basin for use within or without the boundaries of the basin—is thus not a fixed quantity but a variable. Under any given set of conditions it is only necessary that the following general equation be satisfied:

Primary Equation of Hydrologic Equilibrium.—

Surface inflow across the boundaries of the basin		Surface outflow across the boundaries of the basin
plus		plus
Subsurface inflow across the boundaries of the basin	equals	Subsurface outflow across the boundaries of the basin
plus		plus
Precipitation within the boundaries of the basin		Evapo-transpiration losses within the boundaries of the basin

NOTE.—This paper by Harold Conkling was published in January, 1945. *Proceedings*.

[3] (Leeds, Hill, Barnard & Jewett), Los Angeles, Calif.

[3a] Received by the Secretary February 8, 1945.

The magnitudes of the terms on the left side of this equation are substantially independent of any condition or change in conditions within the boundaries of a ground-water basin; consequently, development of a supply of ground water generally requires a change in either the magnitude or the composition of one or more of the terms on the right side of the equation. If the unused surface outflow is decreased, then there can be a corresponding increase in the consumption of water within the basin, or an equal amount can be exported for use elsewhere. In like manner, exportations or greater use of water within the basin can be offset by restriction of nonbeneficial evapotranspiration losses or by reduction of the subsurface outflow. The latter necessarily requires that the ground-water level be lowered within the basin except in the rare instance where subsurface outflow is cut off or reduced by construction of some barrier.

The term "safe yield," as applied to a ground-water basin, has frequently been defined as that quantity of water which can be extracted each year without progressive and permanent lowering of the ground-water level. Although this definition is applicable to alluvial valleys along a stream where the safe yield is limited only by the rate of channel absorption and by the time during which the flow of the surface stream is much greater than the rate of withdrawal from the ground-water supply, it is too restrictive when applied to closed basins and to those where the ground water is not normally near the surface.

In neither of these latter cases could any quantity of water be withdrawn continuously without causing permanent but not necessarily progressive lowering of the ground-water level. The following general definition of safe yield is therefore suggested:

> "The safe yield of a ground-water basin is that quantity of usable water which can be extracted therefrom each year without progressive lowering below the level at which the same quantity can be obtained at reasonable cost."

Determination of Safe Yield.—In general, the determination of the safe yield of a ground-water basin under any set of physical and economic limitations can be made by evaluation of all terms in two primary equations of hydrologic equilibrium: One for the conditions which prevailed in the past and another for the conditions assumed for the future. This method, which evidently is preferred by the author, is necessarily subject to considerable error, because second differences as well as first differences between quantities of relatively great magnitude must be computed.

The problem may frequently be simplified by eliminating all factors which do not affect directly the ground-water supply. The following general equation is then applicable:

Secondary Equation of Hydrologic Equilibrium.—

Absorption of surface inflow within the boundaries of the basin		Subsurface outflow across the boundaries of the basin
plus		plus
Subsurface inflow across the boundaries of the basin	equals	Exportations of water across the boundaries of the basin
plus		plus
Absorption of precipitation within the boundaries of the basin		Evapo-transpiration losses within the boundaries of the basin

Most of the terms on the left side of the secondary equation are subject to less variation than the corresponding terms in the primary equation. For example, although absorption of surface inflow within the boundaries of the basin must equal the difference between surface inflow and surface outflow across these boundaries, it can be computed from measurements of the wetted channel area of the streams and observed rates of percolation. Likewise, it is generally true that the rate of absorption of precipitation is limited and that, when the intensity of rainfall exceeds such observed rates, all the excess passes out of the area in the form of surface runoff.

Direct Methods.—In many cases there are more direct solutions of the problem, which take advantage of the fact that contributions to and losses from a ground-water basin are not characterized by the wide variations typical of other hydrologic phenomena. Such direct methods, described briefly by the author, involve correlation between changes in the ground-water level and observed variations in either the contribution to or the extraction of water from the basin. It is believed that they have much wider application than is assumed by him. The direct method developed by the writer (Case I of the author) may best be demonstrated by assuming, first, a hypothetical case and then showing the result of certain actual determinations of safe yield of ground-water basins. Such a hypothetical case is given in Table 1.

TABLE 1.—HYPOTHETICAL BASIN (AREA 10,000 ACRES)

Successive years	ACRE-FEET		FEET
	Annual contribution less outflow[a]	Annual quantity pumped[b]	Change in water level during year
1	17,000	8,000	+ 9.0
2	15,000	20,000	− 5.0
3	13,000	6,000	+ 7.0
4	17,000	16,000	+ 1.0
5	15,000	22,000	− 7.0
6	13,000	10,000	+ 3.0
7	15,000	18,000	− 3.0
8	13,000	4,000	+ 9.0
9	17,000	12,000	+ 5.0
10	15,000	14,000	+ 1.0

[a] Total contributions to ground-water supply minus total natural outflow vary from 13,000 to 17,000 acre-ft per yr. [b] Pumping draft on ground-water supply varies from 4,000 to 20,000 acre-ft per yr.

In Fig. 7(*a*) the observed change in the average depth to water throughout this hypothetical basin is plotted against the corresponding annual amount of water pumped. The curve which best fits the plotted data is evidently a straight line. The points that fall on the straight line correspond to years in which the annual net inflow equals the average net inflow. The points that fall to the right of the straight line correspond to years in which the net inflow exceeded the average, and the remaining points which fall to the left of the straight line are for those years in which the net inflow is smaller than the average.

The departure of such points from the straight line is equal in each case to the difference between the actual net inflow and the average net inflow. The

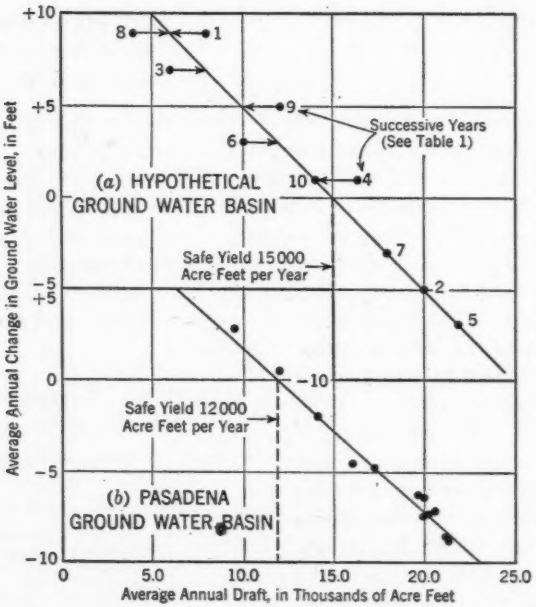

FIG. 7.—DIRECT METHOD FOR THE DETERMINATION OF SAFE YIELD

intercept with the x-axis therefore indicates the safe yield, which in this case is 15,000 acre-ft per yr.

In Table 2 the relationships are given between actual fluctuations of the ground-water level in the Pasadena basin in Los Angeles County, California (Fig. 3), and the quantity of water pumped from that basin from 1922 to 1938. Each point in Fig. 7(*b*) represents the average annual amount of water pumped for five consecutive years in relation to the average annual change in water level during the same five years. Progressive 5-yr periods were taken covering the entire number of years of record to smooth out annual variations in supply.

The area of this basin is 11,900 acres, bounded on the north by a line extending generally east from Arroyo Seco below Devils Gate and Monk Hill.

The area immediately tributary to Santa Anita Wash is excluded, as is the southwest corner lying between Eagle Rock Fault and Raymond Fault.

It is significant in this actual case that the plotted points lie substantially on a straight line and the greatest departure from that line does not exceed 1,000 acre-ft. It must be concluded, therefore, that the net inflow to the Pasadena basin is substantially constant. The straight line intersects the x-axis at 12,000 acre-ft per yr, as compared with the average pumping of about 18,500 acre-ft. Consequently, the safe yield—that is, the difference between the contribution to the ground-water supply and natural outflow and losses from that supply under the conditions which prevailed from 1922 to 1938—was a little less than two thirds of the draft.

TABLE 2.—HISTORICAL RECORD OF PASADENA (CALIFORNIA) BASIN (AVERAGE ANNUAL VALUES)

Successive periods[a]	Quantity pumped (acre-ft)	Change in water level (ft)
1922 to 1927	19,705	− 6.2
1923 to 1928	20,088	− 7.3
1924 to 1929	19,939	− 6.4
1925 to 1930	20,529	− 7.1
1926 to 1931	21,086	− 8.5
1927 to 1932	21,210	− 8.7
1928 to 1933	20,919	− 8.3
1929 to 1934	19,779	− 7.4
1930 to 1935	17,233	− 4.7
1931 to 1936	16,147	− 4.5
1932 to 1937	14,205	− 1.9
1933 to 1938	11,979	+ 0.4

[a] Five-year moving average.

The data pertaining to the Pasadena basin illustrate a single condition of equilibrium. In Fig. 8 the data are plotted corresponding to two different conditions for a ground-water basin located along a major fault in Southern California. Prior to 1927 the safe yield of this small basin was approximately 3,100 acre-ft per yr, but since 1930 an average of only 1,700 acre-ft annually could be withdrawn without progressive lowering of the water level over a typical series of wet and dry years. The shift from one condition of equilibrium to another in the foregoing case was probably caused by some change in natural conditions, presumably by movement along a major fault in 1927.

Similar reductions in the safe yield of the Cortaro basin along the Santa Cruz River in Arizona were disclosed by correlating the amount of pumping from wells with changes in ground-water levels. The land overlying the ground-water basin was irrigated for a number of years, and the return flow from the application of water on the fields reduced the effective draft upon the ground-water supply. Subsequently, farming operations were shifted to another part of the project, and the water pumped from the wells was collected in canals and exported from the basin. The return flow was then limited to leakage from the canals, and lowering of the water level quickly took place. Later, to conserve the supply, the canals were lined with concrete, thereby increasing the net draft upon the ground-water basin. This again resulted in a reduction in the amount of water which could be pumped without progressive lowering of the ground-water level.

Determination of Specific Yield or Area.—The specific yield or the area of a ground-water basin, when either is known, can be determined by the direct method from the slope of the straight line which best fits the plotted points. In the case of the hypothetical basin described in Table 1, a change of 1,000

acre-ft per yr in the rate of withdrawal produced a corresponding change of 1 ft in the average ground-water level. The area of this hypothetical basin is 10,000 acres, so that the gross displacement for a 1-ft change in level was 10,000 acre-ft; consequently, the specific yield (that is, the proportion of drainable voids) was 10%.

In the case of the Pasadena basin the area is 11,900 acres, and the average specific yield of the alluvium in this basin is 9.4%, as measured by the rate of change in level compared with the rate of change in draft.

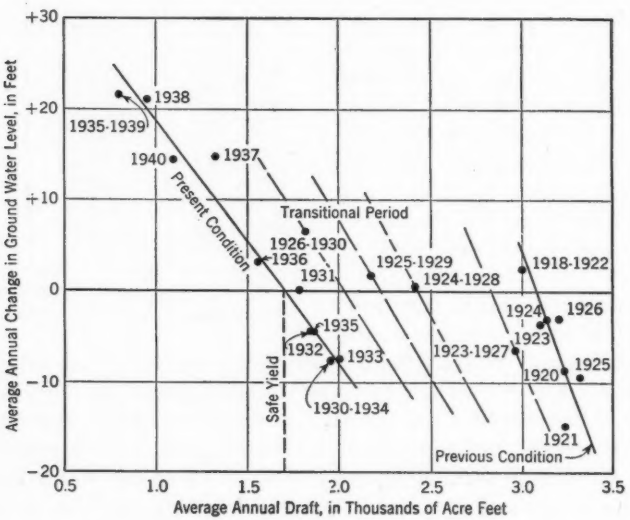

FIG. 8.—CHANGE IN THE SAFE YIELD OF A SMALL GROUND-WATER BASIN

In some cases the actual extent of the ground-water basin, or at least the zone of influence of pumping from the basin, is not known. The effective area can be approximated, however, by assuming a reasonable value for the average specific yield. An extreme case in point was a ground-water basin in Arizona, which was generally thought to extend over only about 1,000 acres. When the quantity of water pumped from this basin was plotted against the corresponding changes in ground-water level, the slope of the curve fitting the points was so flat that the specific yield would have had to be more than 50% if the area of the basin were only 1,000 acres. No natural alluvium has gross voids as large as 50%, and there are few instances in which the drainable voids exceed 15% of the total volume displaced. Obviously, in this case, the ground-water basin extended beyond what had been assumed to be its boundaries, and the effective area of the subsurface reservoir must have been almost 4,000 acres.

Summary.—The direct method of determining the safe yield of a ground-water basin necessarily depends upon the availability of records of performance for that basin under known conditions. However, most of the available supplies of ground water have already been tapped and used sufficiently to

establish the relation between rates of withdrawal and changes in level. Even if it is found on trial that the variations in supply from year to year are such as to make indefinite the draft at which there would be no change in level under average conditions, these variations can be smoothed out by taking periods of time longer than one year. Essentially the method involves only recognition of the fact that continuous pumping in excess of the average net supply to a ground-water basin will produce progressive lowering, and that conversely any reduction in draft will cause the ground-water level to rise to some higher stage of equilibrium.

In the application of the indirect or difference method favored by the author, there is the same need for historical data. Furthermore, these data must be more accurate than those required for direct determination, because of the multiplication of errors resulting from the computation of small differences between large quantities, each subject to observational error.

Where historical records are not available, any determination of the safe yield of a ground-water basin can be only speculative between the reasonable upper and lower limits indicated by experience and sound judgment.

AMERICAN SOCIETY OF CIVIL ENGINEERS

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DISCUSSIONS

UNUSUAL CUTOFF PROBLEMS—DAMS OF THE TENNESSEE VALLEY AUTHORITY

A SYMPOSIUM

Discussion

BY L. A. SCHMIDT, JR.

L. A. SCHMIDT, JR.,[14] M. AM. SOC. C. E.[14a]—The solution of a very difficult cutoff problem in the foundation of a dam under construction has been described by Mr. Hays. This solution reveals the ingenuity used to acquire the desired result, and sets this work apart as unique, effective, and unquestionably economical. Compared with the only other practicable method of attack—namely, open excavation and concrete backfill—the treatment accorded the foundation area under Kentucky Dam described by Mr. Hays is so logical that it is difficult to understand how any one could possibly criticize it. The successful use of large-diameter core drills for excavating purposes on this project appears to justify this type of equipment. In any case, the work done at Kentucky Dam has prevented the development of a foundation problem similar to the one that existed at Hales Bar Dam, as aptly stated by Mr. Black.

Certainly Mr. Hays' application of core drills to excavation would have been the logical and surest method of securing the desired results at Hales Bar Dam had that project also been in the construction stage, or had it been possible or desirable to drain the reservoir. The Hales Bar Dam has been in operation since 1913. However, if these procedures had been used on the project they would have been inimical to the best interests of power production during the critical period before Pearl Harbor, and some other method capable of being executed without draining the reservoir was required at Hales Bar Dam. After considerable study the method described in the paper was evolved.

The writer is particularly grateful to Mr. Fox for his presentation of the geological characteristics inherent in the Hales Bar foundation. The part played by faulting in the development of the zones of weakness against leakage

NOTE.—This Symposium was published in November, 1943, *Proceedings*. Discussion on this Symposium has appeared in *Proceedings*, as follows: February, 1944, by Berlen C. Moneymaker; April, 1944, by Portland P. Fox, and J. K. Black; May, 1944, by L. C. Glenn; and January, 1945, by E. H. Burroughs.

[14] Engr., Chattanooga, Tenn.; formerly, Acting Constr. Engr., TVA, Hales Bar Dam, Chattanooga, Tenn.

[14a] Received by the Secretary January 22, 1945.

in the foundation was probably insufficiently emphasized. Unquestionably, the conditions that either permitted clays and shales from horizontal strata to be washed out by exposure to the differential water pressure, or permitted flows caused by solution of the limestone, were given their initial impetus by faulting as shown in Fig. 13.

The writer subscribes to Mr. Fox's contentions about asphalt, with some reservations. To some extent, Mr. Fox overlooks the effect of temperature on asphalt, and temperature is the relative medium by which the characteristics of asphalt may be judged. In a cavity from 60 to 160 ft below water surface, temperatures of about 56° F are prevalent in the Tennessee River. Thus, when asphalt is placed in the irregular labyrinths (which the channels form) at this low temperature, under sufficient pressure to compensate for shrinkage, it should be capable of long life. Supporting evidence was obtained in the form of asphalt cores removed in drilling, which indicated that asphalt placed twenty years previously had remained in place.

The 1923 placement of asphalt at Hales Bar Dam could have been the final "coup de grâce" to leakage. However, rather too much was expected of asphalt fill at that time since its possibilities had not been exploited to a maximum degree. In addition, an insufficient amount was placed in the widely spaced holes. Had closer spacing been used and had adequate cement grout under pressure been placed for consolidation, it is conceivable that the gains would have been stable.

That asphalt is not as desirable as cement is conceded, but, where cement cannot be placed, asphalt is far more desirable than nothing. Certainly backing up the asphalt with cement is correct and has no definable upper limit.

Mr. Moneymaker principally discusses the part the geologist can play in foundation problems, and the writer agrees with him that the geologist can contribute much in enabling constructors to avoid foundation errors. However, it should be emphasized that the geologist needs cores and data for analytical purposes, and without them his work cannot be effective. The writer wishes to acknowledge the accuracy of the predictions of geologic characteristics made by J. W. Frink, Jun. Am. Soc. C. E., who was stationed at Hales Bar Dam. The location of holes to supplement treatment was determined in many instances from data supplied by Mr. Frink. This type of service, although passive and secondary to the main objective, is often overlooked and Mr. Moneymaker rightly calls attention to its importance.

Final quantities at Hales Bar Dam were:

Description	Quantity
Drilling, in Linear Feet—	
18-in. diameter	34,221
13-in. diameter	7,448
3-in. diameter	55,264
Asbestos-cement liners, in linear feet	30,986
Concrete, in cubic yards	8,653
Reinforcement, in tons	70
Cement grout, in cubic feet	364,361
Asphalt, in cubic feet	135,017

The writer shares the hope expressed by Mr. Glenn that the work done at Hales Bar Dam this time may be more effective and permanent than was the previous work. The visible results at the completion of the job were gratifying. The placement of the final quantities certainly reduced any future potential leakage, and it is doubtful that leakage could again develop to such an extent as it had by 1940. The problem has been of such long standing, however, and the many attempts at its correction have been frustrated so often that the unequivocal assumption that additional maintenance in the future will be unnecessary seems too optimistic. Given careful, orderly inspection and maintenance if and when required, the problem should eventually become history.

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DISCUSSIONS

AMPLIFIED SLOPE DEFLECTION

Discussion

BY WILLIAM C. SPIKER, ARTHUR B. MCGEE, AND JAROSLAV POLIVKA

WILLIAM C. SPIKER,[18] M. AM. SOC. C. E.[18a]—The "traverse" method described in this paper is creative and clever. However, because, as acknowledged, it is limited in application and because, in the writer's opinion, it requires more than the average amount of practice and facility for successful use, it should be simplified and amplified, if possible. Then, too, the writer believes that the questions of signs and "unknowns" in the basic slope-deflection method are not as serious as Mr. Stewart states. It is to be remembered, furthermore, that there are other, more simple methods.

As an illustration, the slope-deflection method can be applied to the case in Fig. 2, with perhaps a slight variation from standard routine. The writer prefers the following sign convention: In the setup equations all θ's have plus signs. The signs for the fixed-end moments refer to the moments which resist the fixed-end moments. They are plus when the resisting moments tend to turn the end of the member to the right and minus when the resisting moments tend to turn the end of the member to the left.

The detailed solution is as follows:

Moment	Computation	Computed moment
M_{AB}	$12\theta A$	$+ 57.18$
M_{AD}	$4\theta A + 2\theta D - 6R$	$- 57.15$
....	$16\theta A = +6R - 2\theta D$	....
Adding and transposing,		
....	$\theta A = +0.375R - 0.125\theta D$	....
M_{DA}	$4\theta D + 2\theta A\ (= 0.75R - 0.25\theta D) - 6R$	$- 42.86$
M_{DC}	$12\theta D$	$+ 142.8 \pm$

NOTE.—This paper by Ralph W. Stewart was published in September, 1944, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: November, 1944, by Leon Beskin; and February, 1945, by John E. Goldberg, and Camillo Weiss.

[18] Cons. Engr., Mountain View, Ga.

[18a] Received by the Secretary January 31, 1945.

M_{DE}	$3\theta D - 3xR$	-100.00
....	$18.75\theta D = +5.25R + 3xR$	....
....	$\theta D = +0.28R + 0.16xR$	-11.91
....	$\theta A = +0.375R - (0.035R + 0.02xR)$	
	$= +0.34R - 0.02xR$	-4.765

Therefore:

M_{AD}	$-4.08R + 0.24xR$	....
M_{DA}	$-4.20R + 0.60xR$	....
....	$-8.28R + 0.84xR$	-100.00
M_{DE}	$+0.84R - 2.52xR$	-100.00
....	$-9.12R + 3.36xR$	0.00

Therefore, $x = 2.715$; $R = +16.67$; and $xR = 45.23$. Then substitute into the basic equations the values of R, xR, θA, and θD and add the moments opposite the equations as shown.

Note that, if the sway in both tiers is the same, the problem is as simple as that for two spans on three supports. The x disappears.

ARTHUR B. MCGEE,[19] ESQ.[19a]—Amplified slope deflection is in reality the traverse method[3] extended to include the displacement of joints, or sidesway. In a normal traverse solution, joint movement is restricted to rotation only. In general, the traverse of a member consists of three sides of known lengths and four angles—namely, a θ-angle at each end and two intermediate Δ-angles. If there is no joint displacement, any two angles are sufficient to determine the remaining two angles; but, if there is displacement of a joint, three angles are required to determine the fourth one. In the latter case, simultaneous equations are usually required.

Mr. Stewart has shown the relationship between the traverse method and slope deflection. The number of essential unknowns in a given rigid frame is the same regardless of the method used for analysis. However, if a method disposes of any of the unknowns in an easy manner and reduces the number of simultaneous equations necessary for the solution of a problem, it merits consideration.

Fig. 4(*a*) shows an unsymmetrical frame with members of uniform moments of inertia. The writer's method of procedure assumes a traverse of the deflected frame (Fig. 4(*d*)). Errors in the direction of deflection angles (Δ-angles) or in the direction of rotation of joints (θ-angles) will not affect the solution. The traverse must be drawn, however, to provide opposing moments at each joint. The assumed traverse is solved and values are found for the Δ-angles. If any angle proves to be negative, the angle was merely assumed in the wrong direction and the numerical value is still correct.

[19] Structural Engr., Bridge and Structural Design Div., Bureau of Eng., City of Los Angeles, Los Angeles, Calif.

[19a] Received by the Secretary February 6, 1945.

[3] "Relative Flexure Factors for Analyzing Continuous Structures," by Ralph W. Stewart, *Transactions*, Am. Soc. C. E., Vol. 104 (1939), p. 521 (see also references therein for demonstrations of the use of the elastic curve traverse).

In Fig. 4(*d*), subscripts have been placed on the θ-angles and Δ-angles in the order of evaluating the angles for ease in following the solution. The Δ-angles are at the third points of the members. The angles of rotation of joint A and joint B are assumed to be θ_1 and θ_2, respectively. By geometry,

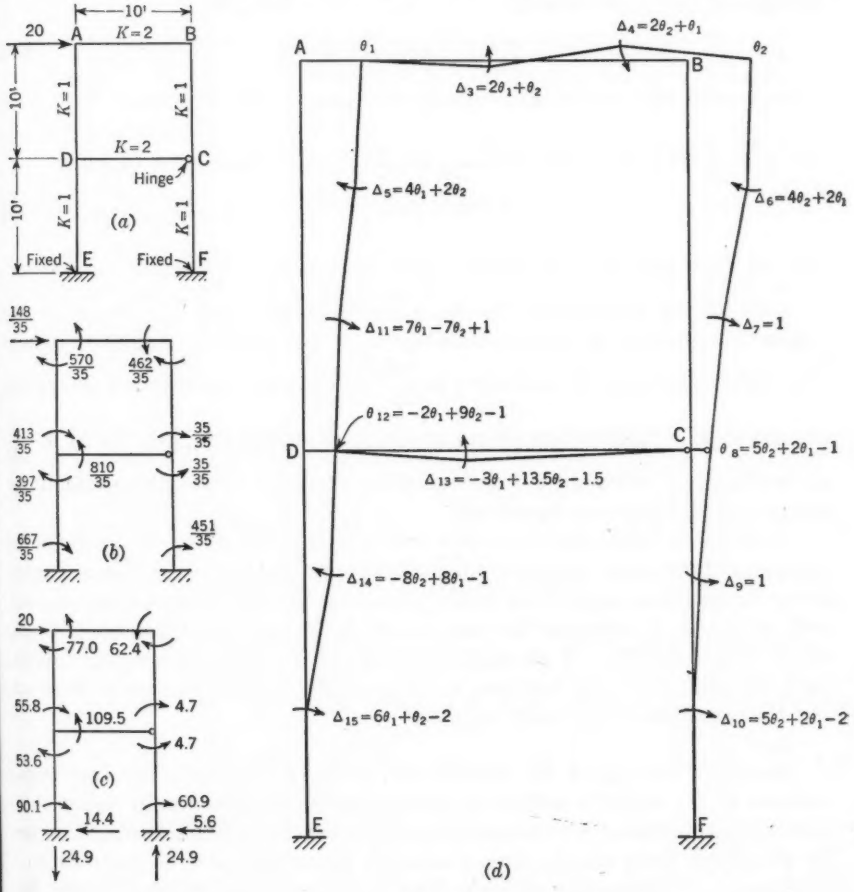

FIG. 4

$\Delta_3 = 2\,\theta_1 + \theta_2$ and $\Delta_4 = 2\,\theta_2 + \theta_1$. Since member AD is half as stiff as member AB, $\Delta_5 = 2\,\Delta_3 = 4\,\theta_1 + 2\,\theta_2$. Likewise, $\Delta_6 = 2\,\Delta_4 = 4\,\theta_2 + 2\,\theta_1$. By assumption, $\Delta_7 = 1$. Then, from joint B to joint C, $\theta_2 + \Delta_6 - \Delta_7 - \theta_8 = 0$; hence, $\theta_8 = 5\,\theta_2 + 2\,\theta_1 - 1$. Since $M_{CB} = M_{CF}$, $\Delta_9 = \Delta_7 = 1$. From joint C to support F, $\theta_8 - \Delta_9 - \Delta_{10} = 0$; therefore, $\Delta_{10} = 5\,\theta_2 + 2\,\theta_1 - 2$. The deflection in member AD must equal the deflection in member BC; so $3\,\theta_1 + 2\,\Delta_5 - \Delta_{11} = 3\,\theta_2 + 2\,\Delta_6 - \Delta_7$, and solving, $\Delta_{11} = 7\,\theta_1 - 7\,\theta_2 + 1$. From joint A to joint D, $\theta_1 + \Delta_5 - \Delta_{11} - \theta_{12} = 0$; and, therefore, $\theta_{12} = -2\,\theta_1 + 9\,\theta_2 - 1$. In the member DC, $\Delta_{13} = \frac{3}{2}\,\theta_{12}$; hence, $\Delta_{13} = -3\,\theta_1 + 13.5\,\theta_2$

-1.5. The deflection in member DE must equal the deflection in member CF; so $2\theta_{12} + \Delta_{14} = 2\theta_8 - \Delta_9$ and $\Delta_{14} = -8\theta_2 + 8\theta_1 - 1$. From joint D to support E, $\theta_{12} + \Delta_{14} - \Delta_{15} = 0$ or $\Delta_{15} = 6\theta_1 + \theta_2 - 2$. About joint D, $\Sigma M = 0$; therefore, $\Delta_{14} + \Delta_{11} = 2\Delta_{13}$; and, substituting the values of the Δ-angles in terms of θ_1 and θ_2:

$$7\theta_1 - 14\theta_2 + 1 = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots (20a)$$

The shear in the second story equals the shear in the first story; therefore,

$$\frac{\Delta_5 + \Delta_{11} + \Delta_6 + \Delta_7}{10} = \frac{\Delta_{14} + \Delta_{13} - \Delta_9 + \Delta_{10}}{10}$$ or, in terms of θ_1 and θ_2:

$$-3\theta_1 + \theta_2 + 8 = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots (20b)$$

Eqs. 20 are solved simultaneously, and they yield $\theta_1 = \frac{113}{35}$ and $\theta_2 = \frac{59}{35}$.

Substituting these values in the Δ-angles of Fig. 4(*d*) and changing the angles to moments by the relationship $M = K\Delta$ give the results shown in Fig. 4(*b*). The shear in each story is $\frac{148}{35}$ which is the applied load producing the moments. Multiplying all the quantities shown in Fig. 4(*b*) by $20 \times \frac{35}{148}$ gives Fig. 4(*c*), which is the solution of the problem. Thus, this problem required two simultaneous equations.

It should be mentioned that, after drawing a logical traverse of the deflected structure, the analyst has numerous choices in starting the evaluation of angles. Some methods are easier than others but no set procedure has to be followed, and, as long as the assumed traverse is solved, the signs will take care of themselves automatically. If the members had had variable moments of inertia, both the stiffnesses and locations of the Δ-angles would have been different, but the procedure would have been the same.

JAROSLAV POLIVKA,[20] M. AM. SOC. C. E.[20a]—There are many interesting features in the author's method of simplifying slope deflection by using characteristic properties of a traverse of the elastic curve. Elastic deformation can be visualized more clearly, and automatic procedures can be reduced to a minimum. However, the author's claim that the method reduces by one half the work of computation required by the slope-deflection method, if applied generally, cannot be upheld. The author's procedure is limited to a few problems, such as the one demonstrated in Fig. 2, which was selected to show the usefulness of the method in a case where a symmetrical two-story frame, subjected to a lateral force, has pin connections with the bases.

It is true that, in the usual slope-deflection method, the joint expressions include terms of the unknown deflections; yet it is not difficult to reduce the simultaneous equations to a general form containing only deflection angles and

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[20a] Received by the Secretary February 7, 1945.

load constants, as has been shown by A. Amirikian,[21] M. Am. Soc. C. E. This reduction is accomplished by substituting deflection values in terms of the deflection angles and the load constants (obtained from the shear equation of each story) for deflections in the joint expressions.

Applying this principle to the author's example in Fig. 2 and referring to Mr. Amirikian's work, the following equations can be written: For joint A:

$$\left(2+1-\frac{3}{4}\times\frac{1}{2}+1-\frac{3}{4\times 2}\right)\frac{A}{2}+\left(\frac{1}{2}-\frac{2\times 3}{4\times 2}\right)\frac{D}{2}=\frac{20\times 10}{2\times 2};$$

or

$$13\,A-D=200 \qquad (21a)$$

Similarly, for joint D, the following equation results:

$$13\,D-A=600 \qquad (21b)$$

Since, in accordance with Mr. Amirikian's procedure, $A=4\,\theta_A$ and $D=4\,\theta_D$, Eqs. 21 become, respectively,

$$13\,\theta'_A-\theta'_D=50 \qquad (22a)$$

and

$$13\,\theta'_D-\theta'_A=150 \qquad (22b)$$

Eqs. 22 contain two unknowns and the solution is just as simple as that obtained by the author's method using the properties of the traverse of the elastic curve. The solution is $\theta'_A=\frac{100}{21}$ and $\theta'_D=\frac{250}{21}$, assuming the relative stiffness $K'=\frac{E\,I}{l}$, or, introducing the basic stiffness used in the paper, $K=\frac{2\,E\,I}{l}$ $\left(\text{ratio of an end moment to the area of the }\frac{M}{E\,I}\text{-diagram which it produces}\right)$, $\theta_A=\frac{200}{21}$, and $\theta_D=\frac{500}{21}$. The moments in the horizontal members are computed from the basic equations; thus: $M_{AB}=2\left(\frac{3}{2}A\right)=12\,\theta'_A=\frac{400}{7}$; and $M_{DC}=2\left(\frac{3}{2}D\right)=12\,\theta'_D=\frac{1{,}000}{7}$. From the shear equation for the symmetrical bent: $M_{AD}+M_{DA}=100$; and $M_{DA}=\frac{300}{7}$. Finally, from the joint equation at point D: $M_{DA}=M_{DC}-100=\frac{300}{7}$.

[21] "Analysis of Rigid Frames (An Application of Slope Deflection)," by A. Amirikian, Government Printing Office, Washington, D. C., 1942, p. 88.

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DISCUSSIONS

WEDGE-SHAPED STRUCTURAL MEMBERS UNDER DIRECT STRESS AND BENDING

Discussion

BY WALTER H. WHEELER

WALTER H. WHEELER,[18] M. AM. SOC. C. E.[18a]—Lucidly and commendably, the author has developed for the use of the profession a comparatively simple set of formulas, for which he does not claim 100% accuracy. However, the formulas are accurate enough for practical purposes, and they are simple.

Although wedge-shaped members are not common in the average type of structure they do, or can, occur frequently in retaining walls (especially high ones), bridge piers, and abutments.

The writer suggests that, in his closure, the author indicate how the analysis could be modified to apply to pyramidal structures.

NOTE.—This paper by Robert B. B. Moorman was published in June, 1944, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: October, 1944, by Phil M. Ferguson, and Stanley U. Benscoter; and January, 1945, by George Winter.

[18] Designing and Cons. Engr., Minneapolis, Minn.

[18a] Received by the Secretary January 26, 1945.

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DISCUSSIONS

STABILITY AND STIFFNESS OF CELLULAR COFFERDAMS

Discussion

BY R. T. COLBURN, A. F. HEDMAN, AND ADOLPH J. ACKERMAN

R. T. COLBURN,[7] M. AM. SOC. C. E.[7a]—It is gratifying to find that the subject of the design of cellular cofferdams is being given the consideration it deserves. This paper is a welcome and comprehensive addition to the data previously existing on this subject, and points the way to possible future developments through field observations on actual structures. The Tennessee Valley Authority (TVA) has designed and built fourteen sheet-pile cofferdams since 1936, and the paper would have been extremely useful in this connection had it been published at an earlier date.

The TVA cofferdams were all constructed of standard 15-in. straight-web piling, with $\frac{3}{8}$-in. web thickness and weighing 38.8 lb per ft. They were driven through varying depths of overburden to solid rock and the cells were of either the clover-leaf type or the circular type with connecting arcs. About 90% of the cells were filled by the hydraulic method and in all cases the material was sand and gravel. Where the rock foundation was unsound grouting was used to consolidate it, to prevent undermining, and to reduce the flow of water under the cells and stop "boils."

The twelve cofferdams used at Pickwick, Guntersville, Chickamauga, Watts Bar, and Fort Loudoun dams, vary in height from 30 ft to 55 ft. The diaphragm or cross-wall type of cell was not used by TVA. The reasons for the selection of the circular and clover-leaf type cells deserve consideration. The designers found by analysis that the saving in amount of piling by use of the diaphragm type decreases as the height increases, and for heights of 40 or 50 ft no appreciable saving results. For the circular cell type of cofferdam for any given length, the amount of piling is practically constant regardless of the diameter of the cell. The circular and clover-leaf cells have a distinct advan-

NOTE.—This paper by Karl Terzaghi was published in September, 1944, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: December, 1944, by G. G. Greulich, and Raymond P. Pennoyer; and February, 1945, by Dean P. Tsagaris.

[7] Constr. Plant Engr., TVA, Knoxville, Tenn.

[7a] Received by the Secretary January 11, 1945.

tage over the diaphragm type if they are filled hydraulically as was the case at TVA. In the diaphragm type the fill in one cell cannot be more than 5 ft or 10 ft higher than the fill in the adjoining cell without distorting the cells. This is especially inconvenient when the cells are filled with a hydraulic dredge. The principal reason for deciding on this type of cell, however, was that any failure due to a split interlock or other cause could be localized in the cell in which the failure occurred. Every cell is self-supporting. In the diaphragm type a serious failure in one cell will cause partial or progressive failure in several adjacent ones.

Both stages of the cofferdams built at Kentucky Dam were 100 ft high and presented a problem of a magnitude on which practically no data existed. Whereas the other twelve cofferdams built by TVA were designed in accordance with the usual current practice for (*a*) bursting of the cells, (*b*) sliding, and (*c*) overturning, it was at once discovered that the factor of vertical shear became extremely important on this high cofferdam. This fourth factor was carefully checked in the Kentucky Dam design and, in fact, it determined the height of the berms inside the cofferdam. The writer agrees with Professor Terzaghi that shear is a factor that should be considered.

To give the greatest interlock strength possible the piling at Kentucky Dam was specified to have a guaranteed ultimate strength of at least 16,000 lb per in. instead of the standard 12,000 lb per in. The increase in value was obtained by more careful inspection and selection of piling being rolled and no change was made in the standard piling section. The cells were driven to solid rock through 40 ft or 50 ft of overburden, which consisted of sand and gravel containing only a small amount of silt and fines and no boulders. The connecting arcs were made with fabricated 30° Y-connections instead of the standard tees and with a longer more flexible arc. This gave much more flexibility and allowed for deviations and inaccuracies in driving the main cells through the heavy overburden. If the usual T-connections had been used the arcs would have been shorter, and this important feature would not exist and undoubtedly many more interlocks in the connecting arc would have been split in driving. It was also felt that by using a 30° Y instead of the standard T-connection the increase in interlock stress in the adjacent piles in the main cell would be less.

Many extensive laboratory tests were made on the fill material to determine the basic characteristics and provide a basis for determination of the design coefficients and assumptions. TVA engineers can readily testify to the accuracy of the author's statement that laboratory tests are inadequate. Several series of tests were made, the best of these showing ranges of 24° to 35° for the angle of internal friction and 0.45 to 0.70 for the coefficient of internal friction. Some tests showed these values to be even higher. The assumed value of this coefficient makes a great difference in the design, especially in the vertical shear strength. The values assumed at Kentucky Dam were 28° 50′ for the angle of internal friction with a corresponding coefficient of 0.55, which are extremely conservative; but in the absence of more conclusive tests a higher

design value did not seem justified. The actual design assumptions for the Kentucky cofferdam were as follows:

No.	Description	Quantity
1	Fill material: Sand and gravel, with a very small quantity of silt	...
2	Overburden: Sand and gravel—in some places a layer of cemented gravel and in one place a 20-ft layer of blue clay	...
	Weight of Material, in Pounds per Cubic Foot—	
3	Dry	110
4	Submerged	65
	Coefficients of Friction—	
5	Gravel on rock (sliding)	0.5
6	Gravel on steel	0.4
7	Steel on steel, in piling interlock	0.3
8	Internal friction	0.55
9	Angle of internal friction for dry and saturated material	28° 50′
10	Slope of berm	1 on 3
	Stresses in Piling, in Kips per Square Inch—	
11	Allowable interlock strength	12
12	Ultimate interlock strength	16
13	Allowable tensile strength in the web	32
14	Ultimate interlock strength in the web	70
15	Factor of safety for sliding, vertical shear, and in the interlock	$\geqq 1.25$
16	Water pressure: Considered the full hydrostatic pressure, to rock	...
17	Active earth pressure: Considered triangular, with the resultant acting at $h/3$ above the base	...
18	Reactive earth pressure: Considered parabolic, with the resultant acting at $h/2$ above the base	...
19	Saturation line: Assumed as shown on the section	...

The plan and section of the Kentucky cofferdam is shown in Fig. 13. The design assumed the cell, with its fill material, to act as a homogeneous mass similar to an earth dam except that its shape was restricted and held in position by the sheet piling. The resistance to sliding, overturning moments, and shear stresses were calculated on this basis, taking into account the effect of the berms, overburden, and the shear resistance of the fill and the interlocks. On this basis the limiting factor in some cases was found to be vertical shear of the material in the center of the cell or at the inside face of the cofferdam. The stress in the sheet piling was calculated from the internal pressure or bursting effect of the material as it was restricted by the sheet piling, plus the stress introduced by the connecting arcs through the Y-pile connections. The effect of complete saturation during the hydraulic filling was considered. On this basis a cell 60 ft in diameter is about the maximum size that can be built without overstressing the interlocks. This was the determining factor in fixing the size of the cells. Obviously, this was too small to take care of the required 100-ft head. The additional support

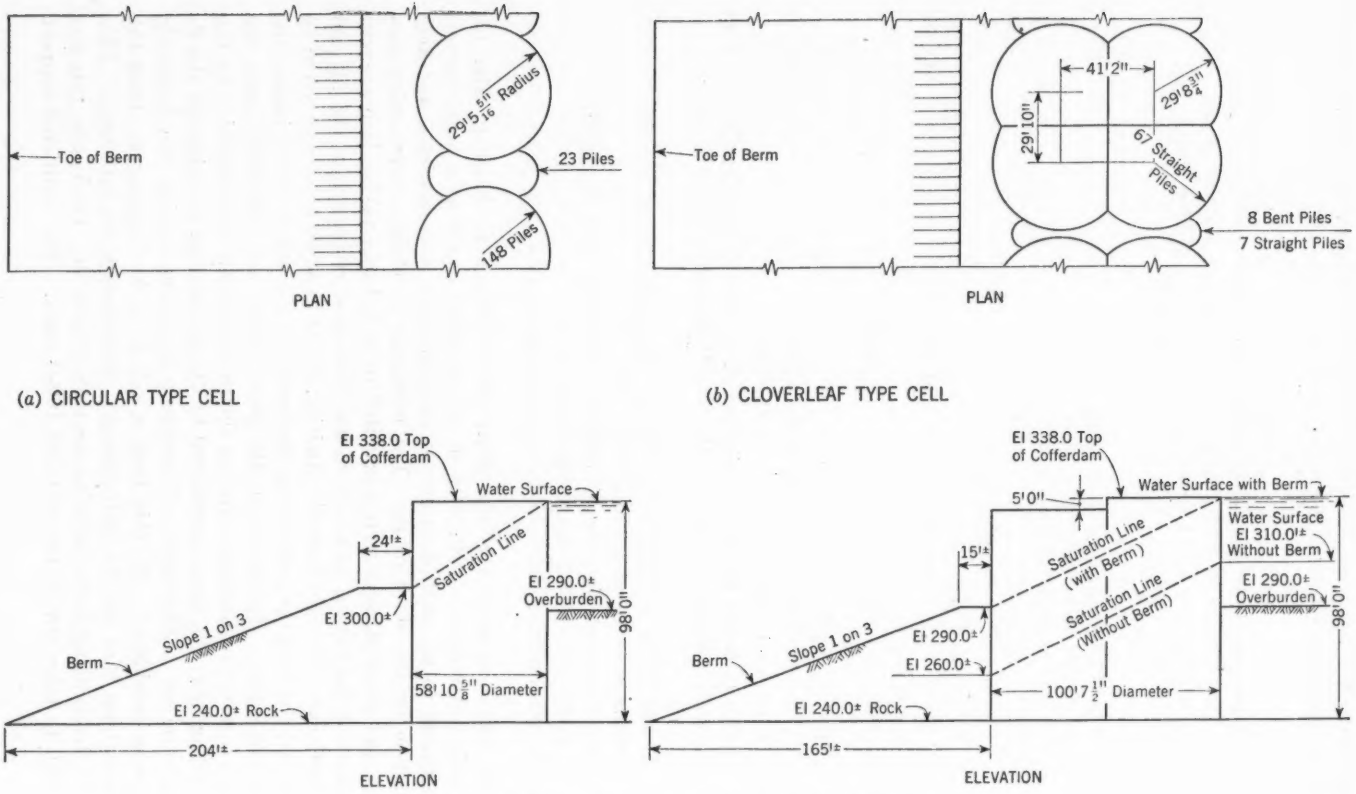


FIG. 13.—PLAN AND ELEVATION OF TVA KENTUCKY COFFERDAM

necessary was then provided by a berm of sufficient height and slope to back up the cells with 100-ft head of water on the outside. It can be seen that the overburden itself provided the major part of the berm which was undisturbed material and this section was used in locations where there was sufficient room for a berm. On the outstream arm, however, the berm on the inside had to be removed for construction reasons. This left a 100-ft-high cofferdam with no berm or overburden on the inside resting on a fairly flat rock surface, or in the second stage on the spillway apron. This is the section where the clover-leaf cell was used which provided a total width of approximately 100 ft. It might be mentioned here that even with this type it was felt necessary to stipulate that the berm could only be removed if the height of the water was 25 ft or more below the top of the cofferdam during the months that the berm was removed. This was done in the dry season and the water remained at a low level so that no harmful results were experienced.

The TVA has had an opportunity to make various tests on their cofferdams and has made several which are of value in that they give actual results under field conditions.

Saturation tests were made at Pickwick and are summarized in Fig. 14. As stated by Professor Terzaghi, the saturation line is almost horizontal except on the outstream third which rises sharply to within a few feet of the water level. Subsequent tests made at Guntersville and Kentucky dams confirm these Pickwick tests and indicate the fallacy of assuming that a cell will ever become nonsaturated. It also shows the extreme importance of proper drainage. All TVA cofferdams were drained by weep holes on the inside face.

At Kentucky Dam 4-in. pipes were inserted in the cofferdam and driven several feet into the cells. Perforations in the pipe provided drainage and the pipes drained into a flume on top of the berm or to a system of corrugated pipe headers laid on the original river bed to keep the water from seeping into the berm itself.

At Chickamauga Dam an attempt was made to determine the stress in the piling by use of a strain gage. It so happened that several cells were built in the dry and filled before they received a water load on one side. Gage points were established at regular intervals on two cells and their connecting arcs and at points about 12 ft apart vertically. Careful readings were taken as the cell was filled and before and after the outside was flooded. The readings were so inconsistent and erratic, however, that no curves could be plotted and no conclusions could be made. Inadequate testing methods might account for this, but the actual conditions in the cells are probably the main reason. For example, it is probable that the piling does not interlock evenly over its full length, and the tension across the face of the piling might easily vary on this account. In fact, if the interlock contact was poor on a certain length of the pile, the tension across the face might be very low at that point with corresponding higher stresses adjacent to it. The only value of this test is that it indicates the very uncertain nature of the stresses in this type of structure and the danger of relying too heavily on a theoretical design.

The TVA also conducted some tension tests on interlock strength of piling which had already been used twice and a third use contemplated. The purpose

FIG. 13.—PLAN AND ELEVATION OF TVA KENTUCKY COFFERDAM

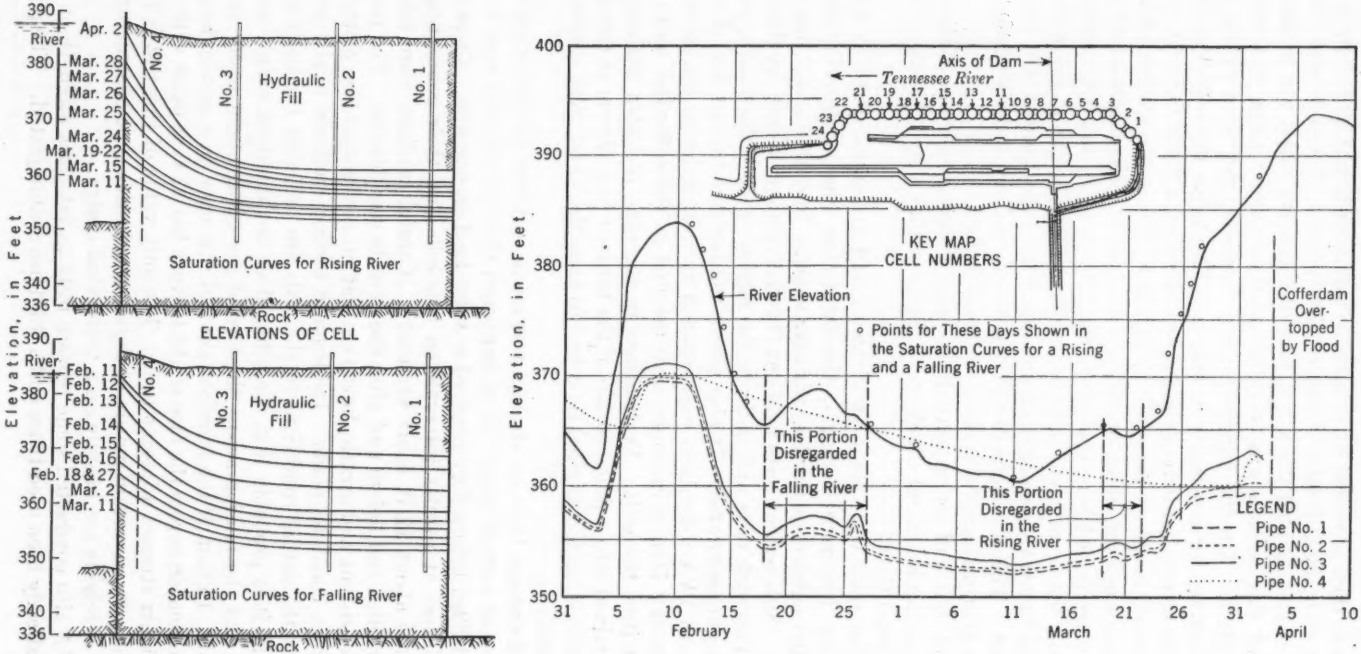


FIG. 14.—SATURATION TESTS; PICKWICK COFFERDAM

of the test was to determine how much loss in interlock strength was to be expected after use. The sections of piling were interlocked at 3° and 22° angles. The specimens were tested to destruction and the results are not very conclusive; but they indicate that, with a certain amount of inspection, piling could be used several times. On TVA construction piling has been used as many as six times.

Deflection measurements were taken at Kentucky cofferdam. The water never came within 25 ft of the top of the first stage and therefore no full height test was available. The measurements taken, however, show so slight a deflection within the range of change in water level of the river as to be impossible to plot with any uniformity. As a matter of fact, one cell for some unexplainable reason showed a slight deflection outward with rising river.

In the second stage cofferdam, however, somewhat better although not conclusive measurements were made. This cofferdam was subjected to water pressure to within a few feet of the top of the cofferdam during the winter and was flooded in April, 1944. The flooding started on March 31, and the following day the water pressure inside the cofferdam equalized the tailwater pressure. On April 2 the upstream arm was overtopped. The maximum river elevation was reached on April 6 when it was several feet above the upstream cofferdam arm. Unwatering started on April 12 and continued through the month. Readings were taken on every other cell between November 6, 1943, and April 24, 1944, to determine the horizontal movement of the top of the cells at various river stages. These readings indicated that the cofferdam cells deflected slightly inward with a rising river, the maximum being near the center of each arm of the cofferdam. The maximum amounted to only 0.13 ft on the upstream arm and 0.095 on the downstream arm. In general, the deflection was less near the shore where the overburden was highest. On the downstream arm the cell nearest the shore deflected outward. Fig. 15 shows the horizontal deflection of the cells in the upstream and downstream cofferdam arms plotted for each cell measured. The deflections decreased after the flooding of the cofferdam, as evidenced by comparing curves Nos. 11 and 12. When, later, the tailwater had dropped below the water inside the cofferdam the cells in the downstream arm deflected outward as indicated by curve No. 13. After most of the water within the cofferdam had been pumped out the cells again deflected inward, as shown in curve No. 14.

The best full-load test on TVA cofferdams occurred on the first one that was built at Pickwick Landing Dam. This was a cofferdam with cells 60 ft in diameter, 55 ft high, and with no berm. It withstood a head of a full 55 ft without any visible signs of distress. At this time the fill was completely saturated, as evidenced by the discharge from weep holes near the top. The only plausible reason why this cofferdam did not fail by vertical shear is that (*a*) the actual coefficient of friction of the material is much higher than laboratory tests indicate, and (*b*) the internal pressure is much higher than active earth pressure, as mentioned by Professor Terzaghi. During filling a certain interlock tension is developed in the piling. It is quite likely that this tension may be maintained by the development of reactive pressure in the

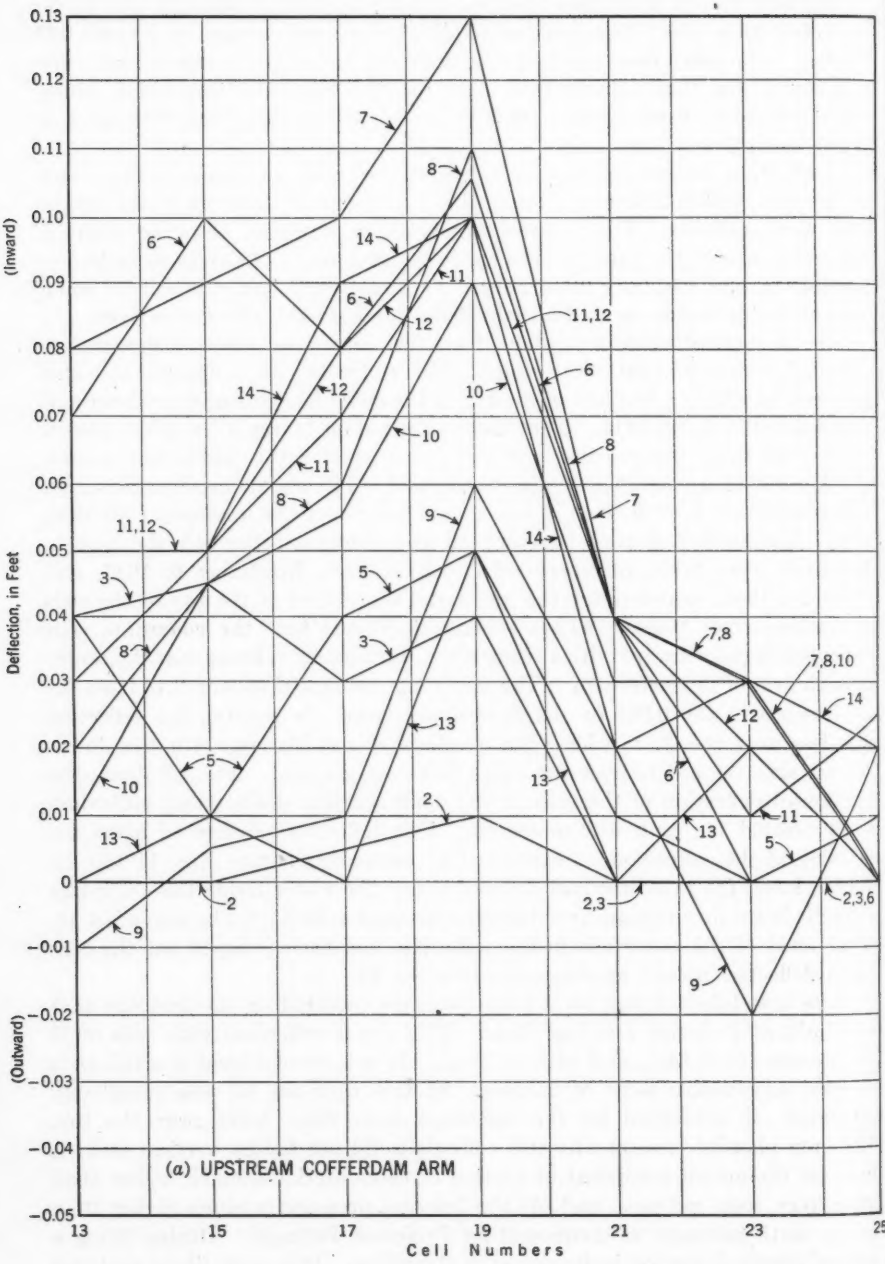
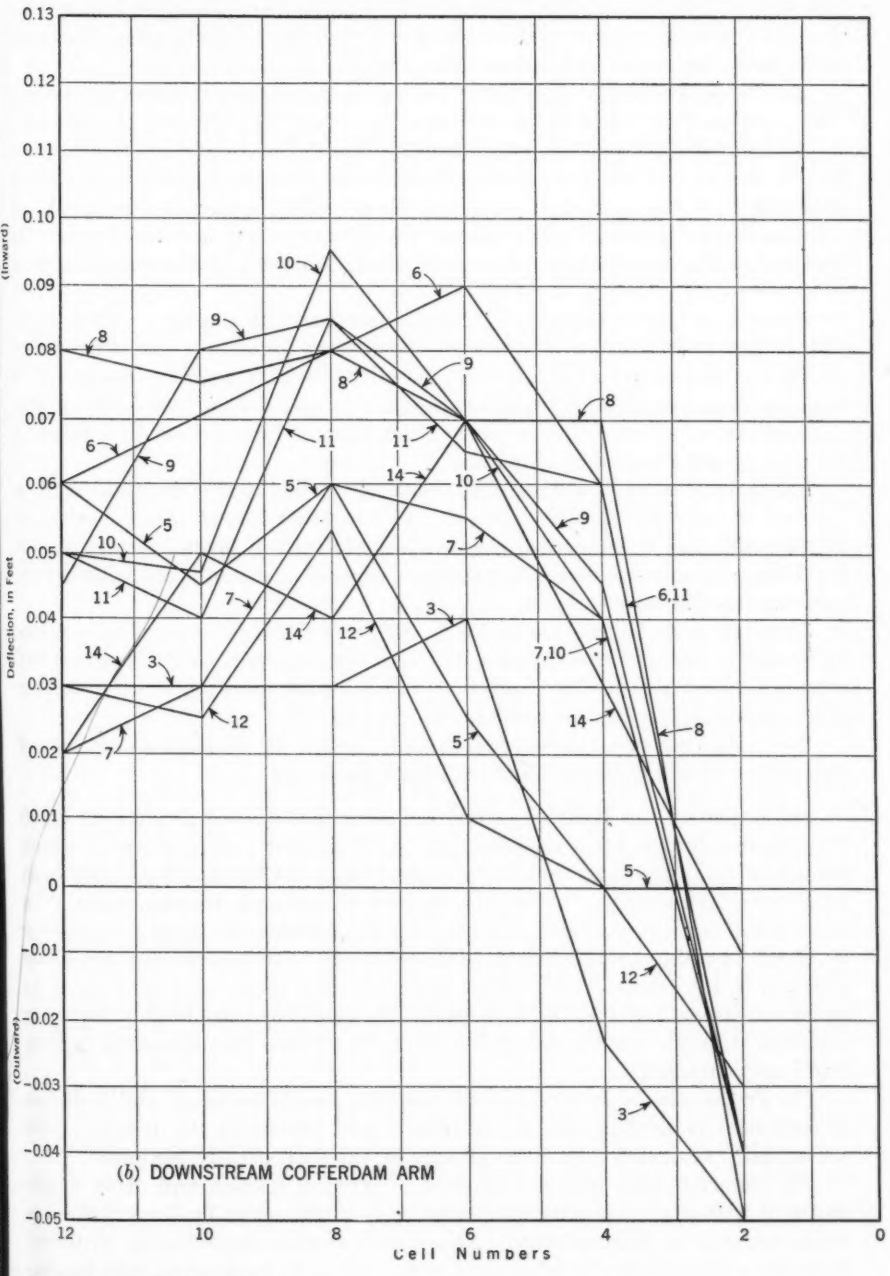


FIG. 15.—HORIZONTAL MOVEMENT



OF CELLS, KENTUCKY COFFERDAM

cell fill. If the cells are filled by hydraulic dredge when the river level is low the piling tension may be quite large and consequently the reactive pressure quite large, which is a definite advantage in favor of hydraulic filling.

In the design of the Kentucky cofferdam the shear resistance in the fill was computed on this basis rather than by using the active earth pressure.

In computing the maximum interlock stress—that is, the interlock stress in the circular cell plus the additional stress due to the connecting arc—it was assumed that the maximum interlock stress which occurred during filling remained even after the cell fill above the saturation line is drained out. In computing the interlock resistance to shear, however, active pressure was used.

Several of the clover-leaf type cells were built in the dry. They constitute the tie-in connection for the second-stage cofferdam and were built inside the first-stage cofferdam and rested on the flat concrete apron. No hard driving was necessary and each pile was seated in a recess left in the apron to act as a key. This is an ideal condition and did not exist for most of the cells which required hard driving.

In some locations at Kentucky Dam the piling was driven through a stratum of cemented sand and gravel. This together with the 50-ft depth produced conditions which caused the interlocks to split in several cases. When the berm was removed these failures were exposed and reinforced with heavy steel bars as the berm was removed.

This interlock failure due to hard driving is the only type of failure experienced on any TVA cofferdam and is easily repaired. This leads to the conclusion that the driving conditions and interlock strength are the danger points in a properly designed cellular cofferdam.

The following general conclusions are possible in connection with steel sheet-pile cofferdam design based on TVA experience:

(1) Experience at Kentucky Dam indicates that driving through 40 or 50 ft of overburden produces excessive stresses in the interlock of a nature which cannot be calculated and that broken interlocks must be expected under such conditions. Dredging to reduce the amount of driving is recommended. To take care of conditions of deep driving, such as at Kentucky Dam, a suggestion is offered to piling manufacturers to design a pile with additional weight and strength in the interlock. It is not enough under these conditions for it to equal the strength of the web in tension. It should be considerably higher in interlock strength to take care of unpredictable stresses encountered in driving into heavy material.

(2) Proper provision for draining the cells should be made and a degree of saturation coinciding with actual tests should be used in the design. Cells are actually saturated to a much greater extent than usually assumed.

(3) Determination of the coefficient of internal friction and other coefficients of friction of the material should be carefully made by tests if there is doubt concerning its suitability. Actual field tests as suggested by Professor Terzaghi are preferable to laboratory tests. If, as in most cases, only laboratory tests are available, they should be used with judgment and conservatism.

The greatest difficulty in the path of a more exact design procedure is the lack of sufficient field test data. This lack is due in some degree to the fact that it is almost impossible to obtain a series of tests or measurements which show any degree of uniformity. It is to be hoped that better and more extensive testing methods can be made on future installations.

The writer wishes to acknowledge the valuable assistance of A. F. Hedman, engineer, TVA, in the preparation of this discussion.

A. F. HEDMAN,[8] ESQ.[8a]—It is with the greatest interest that the engineers of the Tennessee Valley Authority (TVA), who have been connected with the planning and the design of fourteen steel sheet-pile cofferdams on the Tennessee River, have read Professor Terzaghi's logical interpretation of cofferdam behavior. The importance of the paper is not so much that it develops new formulas, as that it endeavors to explain what is happening in a cofferdam, and thus gives the designer a better conception of the factors that will affect the design.

The writer agrees with the explanation of cofferdam behavior, in general, but takes exception to the author's statement about what has and has not been done in the past when designing cofferdams.

Experiences with Cellular Cofferdams.—Admittedly, cellular cofferdams are as reliable as other types and in the writer's opinion are even more reliable. None of the cells in the TVA cofferdams has ever burst or has ever shown any sign of being near failure. With the exception of the cofferdams at Kentucky Dam, no apparent weakness has been discovered in the interlocks even though some piling was used as many as five and six times. At Kentucky Dam steel sheet piling was driven through 50 ft of overburden consisting of sand, gravel, and, under some of the cells, a 10-ft to 20-ft layer of blue clay. After the cofferdam had been unwatered, the overburden was excavated completely to rock at some of the cells, and it was discovered that some of the piling had been driven out of interlock. The failures all occurred near the bottom of the cell and were repaired by tying the piling together, welding heavy steel straps across the break. Where the overburden remained and acted as a berm, such a failure should not be serious because the interlock stress due to pressure from cell fill should be very small or nonexistent.

Deflection of Cellular Cofferdams.—Practically all cells in the TVA steel sheet-pile cofferdams were filled by pumping sand and gravel from the overburden or from near-by deposits in the river. In no case was any appreciable deflection noticed. No doubt this was partly due to the method of filling the cells which caused compaction of the fill material, thus changing the elastic properties. The hydraulic fill also caused higher interlock stresses and therefore greater interlock resistance, which, as the author states, should stiffen the structure. An attempt was made at Kentucky Dam to measure the horizontal deflection for various heights of water pressure after the cofferdam had been unwatered. The deflections were very small, the maximum deflection being

[8] Engr., Constr. Plant Div., TVA, Knoxville, Tenn.

[8a] Received by the Secretary January 25, 1945.

only 0.14 ft. In contrast to the cofferdams described by the author, the crest of the cofferdams at Kentucky Dam did not assume the W-shape after unwatering. This was possibly due to the capability of the ends to deflect.

Permeability of Cellular Cofferdams and of Their Sheet-Pile Enclosures.—As stated by the author, observations were made at Pickwick Dam to determine the saturation line in the cell fill. The TVA has collected some data on the location of the saturation line in several of the cofferdams constructed. Pipes were installed in the cells to serve as observation wells, and frequent readings were taken during rising and falling stages of the river. To provide drainage, weep holes 1.5 in. in diameter were burned on 3-ft centers in every third or fifth pile of the inside row of piling. At Kentucky Dam two rows of nine holes, 4 in. in diameter, were burned in each cell, including the connecting cell, one row at the river bed and one at the top of the berm.

Observation wells in one cell of the lock cofferdam at Pickwick Landing Dam indicated that the saturation line was practically horizontal across the cell. The height of the saturation line above the base varied with the rise and fall of the river. The highest observed level of the saturation line, when the river was almost up to the top of the cofferdam, was approximately 0.71 times the height of river above base of cell. This ratio decreased later, possibly due to either cleaning or adding weep holes.

At Guntersville Dam where seven of the lock cofferdam cells were observed, the saturation line was nearly level across the cells (but only a few feet above the base of the cells), and there was very little fluctuation in this saturation line for variations in the river elevation.

Neither the Pickwick cofferdams nor the lock cofferdam at Guntersville Dam had a berm. The cells in these cofferdams were filled with hydraulic fill consisting of sand and gravel with only a small percentage of silt. In leakage through the cofferdam a comparatively large loss of head occurred at the outside piling. Only a small head was required for flow through the sand and gravel fill. A further loss of head occurred at the inner piling wall. This was quite large in the case of the cell at Pickwick Landing Dam and was probably due to the flow carrying what silt there was in the fill across the cell and depositing it near the weep holes, thus reducing the discharge. At Guntersville Dam there may have been less silt in the fill or the weep holes may have been kept functioning better, possibly by driving rods into them to prevent plugging.

At Chickamauga Dam auger holes were drilled in the third-stage cofferdam cells along the inner piling wall. The saturation line was near the top of the berm used in this cofferdam to back up the cells.

An elaborate drainage system was devised at Kentucky Dam. The cell fill around the 4-in. weep holes was removed and replaced with 3-in. gravel, and the weep holes were then connected by short pipes, welded to the piling, to a flume at the top of the berm and to a system of corrugated pipes at the level of the river bed. Great care was taken to keep all pipes discharging. Observation wells were placed in all cells and the saturation line remained very constant at an elevation of from about 4 or 5 ft below the top of the berm (which was

10 ft above the original river bed), although there was a 32-ft variation in the river level.

All these observations seem to show that it is possible to keep the saturation line quite low, but that it should not be so assumed for design purposes unless great care will be taken to insure that the cell-drainage system functions properly. Where a berm is used, the saturation line should not be assumed to be below the top of the berm. The tighter the interlocks, the less the leakage will be. Therefore, leakage will be affected by the method of filling the cells and by whether piling is new or used.

The author's statement (see heading, "Part I. Practical Considerations: Permeability of Cellular Cofferdams and of Their Sheet-Pile Enclosures"), "The seepage across a cofferdam with such a foundation merely affects the tension in the locks," is contradictory to his subsequent statements that the amount of saturation affects the sliding resistance of a cell.

Cellular Cofferdams on Rock; Sliding Resistance.—The average width of TVA cofferdams has been 1.60 r. The lower value is due to the use of smaller connecting cells. As stated by Professor Terzaghi, the sliding resistance at the base of the cell should be found by taking into account the saturated state of the fill. He implies, however, that this should be done only where the rock is smooth. The writer does not agree that the unit weight of the fill in a moist state should be used in Eq. 3. The weight per cubic foot of saturated fill at Kentucky Dam was approximately 18 lb more than that for the same volume of moist fill. Where H_s is large, a considerably higher value for F_{bs} will result if the weight of saturated fill is used. As correctly stated by Professor Terzaghi, the saturated state of the fill was not considered at Pickwick Dam when investigating for sliding. On subsequent jobs saturation was taken into account and a berm added on the inside not only to obtain sufficient resistance to sliding, but also to keep the resultant within the middle third. At Pickwick Dam the rock surface was fairly smooth, and considerable resistance against sliding was apparently obtained by the steel sheet piling being driven into rock.

Cellular Cofferdams on Rock; Resistance Against Internal Shear.—Contrary to the author's assumption, the possible failure of a cofferdam due to excessive vertical shear through the fill and the piling has been considered by others. Lengthy discussions on this and other related subjects have taken place among TVA engineers especially before the final design of the Kentucky cofferdam was begun. The writer feels that a summary of the methods used by TVA engineers when designing the cofferdams at Kentucky Dam will show that:

1. Uplift was considered to be based upon an assumed saturation line;
2. Vertical shear was computed and resistance to vertical shear was calculated by assuming an internal pressure as stated later (the interlock resistance was taken into account); and
3. The interlock stresses were computed first by considering the pressure that existed during hydraulic filling, and second, after unwatering, by including the water pressure within the cell corresponding to the height of the saturation line.

The Kentucky cofferdam which consisted of both circular and clover-leaf type cells has been described fully elsewhere (34)(35).[35b] A brief summary of the design assumptions used, and the design procedure follows.

Fig. 16 shows a typical cross section through the upstream or the downstream arm of the cofferdam. Circular cells with diameters of 58 ft 10 in. were selected. Piling was driven to rock and the cells were filled hydraulically. Before unwatering, the overburden was dredged to leave a berm as indicated, and an additional 10 ft of fill was added to bring the top of the final berm within 38 ft of the top of cofferdam. An equivalent cofferdam width was assumed equal to the combined areas of one circular cell and one connecting cell, divided by the distance between main cells. The saturation line for two conditions of berm drainage is shown in Fig. 16. Other design assumptions were:

No.	Description	Quantity
1	Water pressure, in feet, assumed acting on the full height of the entire structure	98
	Weight of Material, in Pounds per Cubic Foot—	
2	Dry or drained	110
3	Submerged	65
	Coefficients of Friction—	
4	Gravel on rock	0.5
5	Gravel on steel (corresponding to $\delta = 21° 50'$)	0.4
6	Steel on steel	0.3
7	Coefficient of internal friction, tan ϕ, for dry or saturated material ($\phi = 28° 50'$)	0.55
	Allowable Stresses in Sheet Piles—	
8	Interlock tension, in pounds per linear inch	12,000
9	Web tension, in pounds per square inch	32,000
10	Section modulus (feet^3) based on the average width of cell	391
11	Factor of safety against interlock failure, sliding, and vertical shear (factor of safety against overturning not computed)	1.25

The active earth pressure was computed by the Coulomb formula,

$$P_{e,a} = \frac{C_1 \gamma H^2}{2} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (40)$$

For level surfaces, $\phi = 28° 50'$; $\delta = 21° 50'$; and $C_1 = 0.29$. The pressure distribution for active earth pressure is triangular, the resultant acting at a point distant $H/3$ above the base.

The reactive earth pressure, for broken surfaces, was computed graphically. The pressure distribution is parabolic, with the resultant acting at a point distant $H/2$ above the base.

Computation of Interlock Stresses.—Maximum stress occurred during filling by hydraulic dredge when the fill was saturated to the top of the cell.

[35b] Numerals in parentheses, thus: (34), refer to corresponding items in the Bibliography (see Appendix I of the paper), and at the end of discussion in this issue.

FIG. 16.—DESIGN ANALYSIS, KENTUCKY COFFERDAM

Tension was computed at the top of the overburden on the basis of triangular distribution of pressure. Actually, maximum tension will occur at a higher elevation due to the restraint of the overburden. The increase in interlock stress due to the connecting arc was considered (discussed subsequently).

Stability of Cells.—The following cases were analyzed:

1. Circular cells: (*a*) Berm saturated; and (*b*) berm completely drained.
2. Clover-leaf cells: (*a*) Berm saturated; (*b*) berm completely drained; (*c*) berm removed; (*d*) clay seam at the base with berm saturated; and (*e*) clay seam at the base with berm removed.

Résumé of Procedure for Determining Interlock Stresses.—Only the design of the circular cells with the berm saturated or completely drained will be discussed in this résumé. When investigating the interlock stresses that might occur during filling, it was assumed that the river would not be lower than

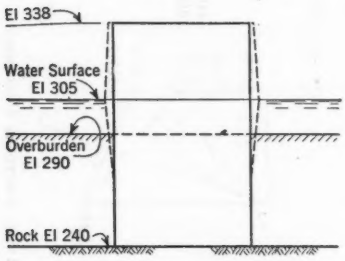


FIG. 17.—SECTION THROUGH CELL DURING FILLING

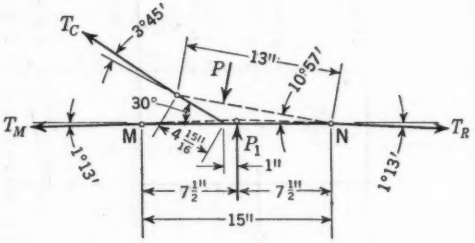


FIG. 18.—DETAIL DATA FOR A 30° WYE

El. 305 (see Fig. 17). During filling, the pressure adjacent to steel sheet piling at El. 290 (in pounds per square foot) would be from:

Water pressure within the cell.....	$62.5 \times 48 =$	3,000
Submerged fill in the cell.....	$0.29 \times 65 \times 48 =$	904
		3,904
Outside water pressure.....	$-62.5 \times 15 =$	− 937
		2,967

In the main cell alone the interlock stress will be $\frac{2,967}{12} \times 29.44 = 7,280$ lb per lin in.; and in the connecting cell it will be $\frac{2,967}{12} \times 9.57 = 2,365$ lb per lin in.

The pressure of the water and the submerged cell fill, of course, changed the shape of the cells into the familiar barrel form. As the water drained out of the cell fill, the cell probably maintained most of its changed shape, the reactive earth pressure being substituted for the water pressure as the cell tended to contract. For this reason it was felt that the interlock tension would not be reduced appreciably as the cell fill drained. Therefore, when computing the maximum interlock tension, the value obtained in the main cell

alone was used, to which was added the effect of the connecting arc. To reduce this effect a 30° wye was used instead of the more common 90° tee.

The forces acting upon the wye are shown in Fig. 18. Considering a section of piling, 1 in. high, the following forces were known: $T_c = 2{,}365$ lb; $T_m = 7{,}280$ lb; and $P_1 = 1.25 \times 2{,}967 \times \frac{1}{12} = 309$ lb.

By projecting the forces on line MN, Fig. 18, and on a line perpendicular to line MN, it was found that $P = 1{,}292 \text{ lb} > \frac{2{,}967}{12} 1.08$, indicating that reactive pressure was developed. Knowing P, it was then possible to find the maximum interlock stress T_r. This stress was found to be equal to 9,500 lb—more than 50% higher than would have been the case if the author's suggested empirical formula, Eq. 28*f*, had been used. The method of filling the cell and the river level at time of filling should definitely affect the interlock tension, making it impossible to establish and use Eq. 28*f* as suggested by Professor Terzaghi. The interlock stress on all TVA steel sheet piling cofferdams (including the ones at Pickwick Landing Dam) was computed by assuming water pressure and saturated fill within the cell—for the full height of the cell—counteracted by the water pressure (low river) acting on the outside of the cell. The factor C (the ratio between horizontal and vertical pressure) in most cases was close to 0.7.

The interlock tension was also computed for the condition after unwatering, and was found to be smaller than the foregoing value.

The average width for the aforementioned cofferdams was approximately 0.89 H for the first ones designed (cells assumed drained and no berm used) and varied between 0.50 H and 0.79 H for those with berms (saturation and vertical shear considered).

Professor Terzaghi states, in the second paragraph following Eq. 21, that

> "If the inner side of a cellular cofferdam is supported by a berm whose height is almost equal to the height of the dam, both the cross-walls and the inner row of sheet piles are acted upon by equal earth pressures from both sides. This condition practically eliminates the tension in the locks, but it also eliminates the lock friction."

This statement is true if the cofferdam is filled while the river is near the top of the cells, which apparently was the case for the cofferdams in New York, mentioned by the author. However, if the cells are driven and filled hydraulically with the river at a low level (which is generally the case where dams are constructed), a large pressure is created in the cell fill, resulting in a high interlock stress. This pressure and the corresponding interlock tension are likely to be maintained by the reactive pressure induced in the cell fill by any tendency of the piling to contract. A berm added afterward on the inside of the cofferdam will exert a pressure far smaller than the pressure within the cell. With rising river the berm may develop reactive pressure, and it is possible that the interlock tension may be somewhat reduced.

Résumé of Procedure for Determining the Stability of Cells.—A saturation line was selected extending from the top of the cell on the river side and sloping

down to the top of the berm on the dry side. Perhaps it might have been just as well to select a horizontal saturation line about 20 ft below the top of cell. Observations made after the cofferdam was completed indicated that the saturation line at the inner piling wall was actually below the top of the berm (see also "Part I. Practical Considerations: Permeability of Cellular Cofferdams and of Their Sheet-Pile Enclosures"). Not knowing where the saturation line in the berm might be, the TVA engineers decided to investigate the cell with the berm being either saturated or completely drained.

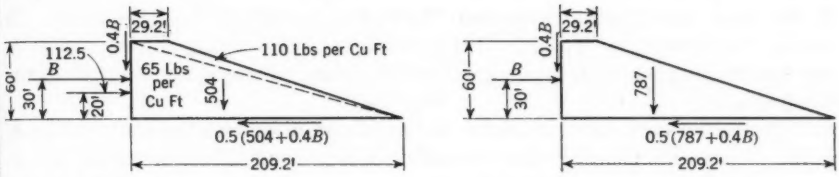
The weight of the cell and the berm, as well as the forces acting on the cell, was then determined. When computing the weight of the cell and the berm, uplift, as measured by the distance from the assumed saturation line down to rock, was taken into account by using the submerged weight of fill. The following results for a 1-ft length of cofferdam were obtained:

No.	Description	Quantity
	Weight of Material, W, in Thousands of Pounds, or Kips—	
1	Cell fill (piling not included), W_c	349.7
2	Saturated berm, W_b	504.0
3	Completely drained berm, W_b	787.0
4	Center of gravity of the cell fill (distance in feet from the center line of the cell)	0.95
	Forces, in Kips, Acting on the River Side of the Cell—	
5	Water pressure on the full height of the cell ($P_1 = 0.5 \times 0.0625 \times 98^2$)	300.0
6	Pressure from overburden; saturated ($P_{e,a} = 0.5 \times 0.065 \times 0.29 \times 50^2$)	23.5
7	Vertical friction on the cell; downward ($F_1 = 23.5 \times 0.4$)	9.4
	Forces, in Kips, Acting on the Dry Side of the Cell; Saturated Berm—	
8	Water pressure ($P_2 = 0.5 \times 0.0625 \times 60^2$)	112.5
9	Maximum sliding resistance of berm; see Fig. 19(a) [$F_b = 0.5\ (504 + 0.4\ F_b)$]	286.5
10	Reactive pressure by graphical construction	362.0
11	Maximum vertical friction on the cell ($F_2 = 0.4\ F_b = 0.4 \times 174$)	69.6
	Forces, in Kips, Acting on the Dry Side of the Cell; Drained Berm—	
12	Water pressure	0
13	Sliding resistance of berm; see Fig. 19(b) [$F_b = 0.5\ (787 + 0.4\ F_b)$]	492.0
14	Reactive pressure by graphical construction	561.0
15	Maximum vertical friction on the cell ($F_2 = 0.4\ F_b = 0.4 \times 492$)	196.8

The maximum reactive pressure of the berm was found, by graphical solution, to be greater than the sliding resistance of the berm. The latter, therefore, became the controlling factor, and was the maximum resisting force that

could be developed if necessary. The horizontal forces, water pressure, and overburden pressure, which act on the river side of the cell will be resisted by the friction between the cell and the rock and by the berm. In the case of a saturated berm, water pressure between the cell and the berm will help to counteract any movement or deflection of the cell.

Next, it was assumed that full friction was developed under the cell, and that the berm then had only to develop a resistance equal to the difference



(a) SATURATED BERM (b) COMPLETELY DRAINED BERM

FIG. 19.—FORCES ACTING ON THE BERM

between the horizontal forces on the river side and the friction F_c under the cell, where $F_c = 0.5\,(W_c + F_1 - F_2)$. This resistance was found to be considerably smaller than the sliding resistance F_b of the berm. The resistance, B (Figs. 16 and 19), was applied at one half the height of the berm. The vertical force $F_2 = 0.4\,B$ was computed, and, by taking moments of all the forces about point O at the toe of the cell, the pressure at the base of the cell was calculated. This pressure was assumed to vary as a straight line. Next, the vertical shear in the cell was determined. On any vertical plane in the cell this shear would be equal to the resultant of all vertical forces to the left or the right of the plane considered—that is, the sum of F_1 plus the weight of cell fill W_c (partly dry and partly saturated) less the pressure on the base. In the computation of the shear resistance of the cell fill, the pressure within the fill was assumed to vary as a straight line between the pressure of water and overburden, $P_1 + P_{e.a}$, on the river side to the resistance from berm and water, $B + P_2$, on the dry side. By multiplying the pressure by 0.55, the assumed coefficient of internal friction, the resistance to vertical shear was obtained. The resistance to failure in vertical shear which exists in the piling interlock was added to the shear resistance of the cell fill. Although it was felt that the high interlock stress, which occurred during the hydraulic filling of the cells during low water in the river, would remain after unwatering and subsequent draining of the cell fill regardless of variations in river elevations, it was decided to be conservative because there was no proof that such an assumption had been made before. The resistance of the steel sheet piling to vertical shear was consequently based on the interlock stress produced by active pressure. Another reason for doing this was the possibility that, with maximum water pressure on the river side and the corresponding resistance from the berm, the cell fill would possibly be squeezed together somewhat and thus would reduce the initial interlock stress obtained during hydraulic filling. The resistance of the piling to shear was determined by multiplying the interlock stress by the coefficient of friction in

the interlocks. By combining the resistance in the cell fill and the resistance in the piling interlock, the total resistance for the cell was obtained.

If this resistance is less than the shear in any vertical plane a-b through the cell (see Fig. 16), the cell will fail in vertical shear and will deflect farther toward the berm. This movement will increase the resistance of the berm until equilibrium is obtained and the horizontal movement is stopped. The increased berm pressure will decrease the vertical shear and increase the resistance to vertical shear as indicated by the curves in Fig. 16. The resistance B created in the berm cannot be greater than the sliding resistance F_b of the berm. It cannot be greater than the sum of the horizontal forces acting on the river side; nor can it be expected to be so great that the heel pressure is greater than the toe pressure.

For the cofferdam at Kentucky Dam the minimum resistance of the berm, required to prevent sliding, was not sufficient to prevent failure in vertical shear, if the berm should be saturated. The maximum resistance $B = 174.0$ kips (plus 112.5 kips of water pressure) for the saturated berm and a resistance $B = 259$ kips for the drained berm (which gave uniform pressure at the base of the cell) both proved to produce sufficient resistance to failure in vertical shear. Smaller berm resistances—203 kips for the drained berm and 142 kips (plus 112.5 kips of water pressure) for the saturated berm—were required to give zero heel pressure.

A similar procedure followed when designing the clover-leaf cells is not discussed herein.

Cellular Cofferdam on Rock; Without Overburden and Without Berm.—To form an idea of the stability of a cofferdam built on rock, where no overburden exists and no berm is used to support the cells, the writer investigated the possibility of a cell in such a cofferdam failing by sliding on the rock or in vertical shear. The following assumptions were made:

(*a*) The cell would be filled hydraulically with sand and gravel, water being present outside the cell not higher than halfway up the cell;

(*b*) The pressure produced inside the cell would remain, due to the tendency of the cell to contract and create reactive pressure, and no additional increase would occur after the water on the outside had risen to the top of the cell (after unwatering of cofferdam);

(*c*) The interlock tension during filling would remain after unwatering and subsequent draining of the cell, regardless of outside water pressure;

(*d*) The saturation line after unwatering would not be higher than halfway up the cell; and

(*e*) The coefficient of friction against sliding on rock and the coefficient of internal friction would be 0.50 and 0.55, respectively (both values conservative). The coefficient of friction in the interlock was assumed to be 0.3.

The following safety factors were found for various dimensions of cells:

Width of cell	Against failure by sliding	Against failure in vertical shear
0.85 H	1.19	1.29
1.00 H	1.40	1.79

If the cells are filled during a period when the water level is lower than that for the foregoing assumption, the cofferdam should be still safer. The opposite would be true if the water level were higher.

Had the cells in the cofferdam been filled with dry material, the internal pressure during filling would have been considerably smaller. After completion of the cofferdam and unwatering, the outside pressure would possibly increase the internal pressure somewhat and thus the resistance to shear, but a cell built in this manner would always be weaker than one filled hydraulically.

As stated by Professor Terzaghi, the internal pressure in the cell is not known definitely. Several assumptions are possible. Whether the cell is filled hydraulically or with dry fill, the horizontal pressure on the inner and the outer walls can be determined. If, next, the pressures on the base and at various other levels above the base, due to overturning moments, were computed, another horizontal pressure diagram could be determined. Should the greater value found by these methods be used when investigating vertical planes on either side of the center of the cell? Does the external water pressure increase the pressure in the fill at the outer wall as assumed in the Kentucky cofferdam design? Answers to these questions would do much to clarify the designer's problems.

The writer is grateful to R. S. Blomfield, associate civil engineer, Construction Plant Division, TVA, for his valuable assistance in the preparation of this discussion.

ADOLPH J. ACKERMAN,[9] M. AM. SOC. C. E.[9a]—This paper represents an important milestone in the history of retaining wall structures composed of sheet piling cells filled with sand and gravel. According to past practice, it has been customary to regard such structures as gravity walls which were designed under two very elementary considerations. First, the width of the wall should be approximately equal to from 0.83 to 1.0 times the head of water, in feet, which the wall is to support; and second, the cylindrical or hoop stress in the cells and in the interlocks should not exceed a designated acceptable value. Many sheet-pile cellular cofferdams have been constructed under these principles, and experience has demonstrated that such cellular structures, when filled with sand and gravel, are reasonably successful for cofferdams and other temporary types of construction.

In spite of the fact that the foregoing statements provided sufficient criteria for the design of sheet-pile cellular walls, there was, nevertheless, in practically every case, a large amount of scientific-appearing computation work performed to make it seem that the structure had the benefit of a careful design analysis. In many cases, however, the assumptions, under which the mathematical analyses were developed, were at complete variance with the actual conditions surrounding the structure. The reassuring thing with respect to past practice has been the fact that, in the end, if the width was approximately equal to from 0.83 to 1.0 times the height of the wall, if the structure stood on a suitable

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[9a] Received by the Secretary February 9, 1945.

foundation, and if it was filled with a stable granular material, the wall could be considered safe and the computations a success.

Professor Terzaghi's paper summarizes the following significant points:

1. It is incorrect to assume for design purposes that a sheet-pile cellular wall acts like a solid gravity wall;
2. The cellular wall is a composite construction of two different materials—steel and soil (the stresses in each material and the interaction of the two materials are analyzed);
3. The distribution of the stresses in the cell fill material is clarified in terms of basic principles of soil mechanics;
4. The frictional stress of the earth fill on the inside of the steel cell is defined;
5. The principal stability of the structure is obtained from the cell fill, with the steel shell acting as an important stiffening influence;
6. High frictional resistance in the interlocks is desirable;
7. In some cases the vertical shear in the cell fill may be a point of critical weakness (in the past, this generally has been ignored in the design);
8. Percolation under cellular walls resting on soil foundations requires close attention;
9. The permeability in sheet-pile cellular walls is clarified; and
10. It is a fallacy to assume that cells with weep holes in the back-wall drain to the bottom of the weep holes. (This assumption in the past has been intended to assure reduced interlock stresses, and maximum weight of cell fill for stability.) The buoyancy effect which results when cells are not adequately drained reduces the stability of the cells.

Professor Terzaghi states that a cellular cofferdam consists of two very different materials—steel and soil—and that its elastic properties can be compared to those of a composite construction material such as reinforced concrete. Although the composite construction can be described readily in this manner, it appears preferable to identify the sheet-pile cellular wall as a "plastic structure," to which the usual laws of elastic structures do not apply except in special instances where a cementing action in the cell fill material might result in the development of typical characteristics of elasticity. For example, when the writer was studying means for increasing the stiffness of a cellular wall which started to deflect excessively, he considered the possibility of welding together all the interlocks in a zone about 10 ft high in the upper part of the diaphragm cross-walls. However, in discussing this subject with the writer, Raymond P. Pennoyer, Assoc. M. Am. Soc. C. E., promptly showed that this would be a mistake because it would introduce a rigid section into a plastic structure. In the event of further deflection, the upper section of the diaphragm could not adjust itself with respect to the body of the structure, and there would be a localized building up of stresses in the welded area which might eventually result in sudden failure.

Professor Terzaghi has presented theories and formulas which are intended to be more rational than the methods of analysis which have been used. Fortunately, in the past, good judgment has prevailed sufficiently so that the

mathematical analyses were not relied upon to an extent that might have been harmful. However, the paper is timely because the possibilities in connection with cellular structures are rapidly expanding and several recent installations in so-called permanent structures have opened up new fields for the utilization of sheet-pile cellular construction. For such structures, it is important to have more precise and dependable means of design.

A few engineers with considerable experience in this class of structure have advocated the use of the middle third theory of stability analysis. In the hands of these engineers this may be an acceptable check or yardstick, but it is a fair presumption that considerably more reliance probably is placed on judgment than on computation. The application of the middle third theory to plastic structures reminds one very much of the housewife who has learned to determine when a flatiron is hot enough for ironing by moistening the fingers and for an instant tapping the hot spot. With the requisite skill and judgment, such a test serves its purpose, but in less experienced hands it may lead to burning the fingers!

The chief objection to the arbitrary use of the middle third theory is that it tends to classify structures of the cellular cofferdam type with the rigid type of concrete structures, such as gravity dams. For most engineers the design of a cellular structure is an infrequent incident, and, when such an assignment develops, there is a natural tendency, particularly among the younger designers, to apply, automatically, their usual training in the theories of elasticity and gravity structures without developing a proper concept of the distribution of stresses as they actually occur in a cellular cofferdam. On a number of occasions the writer has inspected design computations for the stability of cellular structures which disclose this tendency of blissfully accepting the middle third theory for stability. In some cases design assumptions were set up for applying the middle third theory in a manner which indicated a complete lack of concept regarding the actual distribution of stresses. It may be that some of this faulty reasoning accounts for the fact that as recently as 1944 it is still possible to hear about a failure of a cellular sheet-pile structure.

Such failures, of course, will continue to persuade conservative engineers to regard cellular type of construction as useful purely for temporary purposes such as cofferdams. It will only be through such attempt as that presented in this paper, in developing a more rational concept of stress distribution in such structures, that broader acceptance will be accorded this type of construction for so-called permanent structures.

In view of the potentially larger use of steel sheet piling, once this type of construction wins proper acceptance for permanent structures, a paper such as this should be a sign of encouragement to the manufacturers of sheet piling. The author's painstaking effort to remove some of the mystery, and to develop a clear explanation of the principles of soil mechanics as they apply to this type of structure, deserves strong encouragement.

In the past, the best-known reservoirs of accumulated knowledge in this field have been with the manufacturers of sheet piling. Much valuable guidance and advice are available from such sources, but it seems only proper to point out that engineers and builders with limited experience in this field will

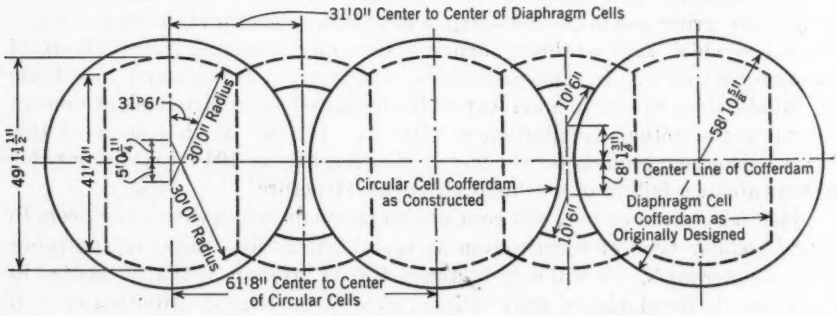
find themselves in an untenable position, in the event of failure, if they assume that the manufacturer accepts any responsibility for the design of the structure, or any obligations or liability beyond those covered by the manufacturer's guarantee in the fabrication of piling. Under the circumstances, any cellular sheet-pile structure of importance should have the benefit of the best obtainable professional engineering advice. This is particularly so for cases where there is the added factor of a difficult foundation condition.

The most elaborate design theories are of little use, however, if proper construction methods are ignored. Since a considerable portion of most cell structures is driven below ground or below water, where it cannot be seen, there is a serious possibility of wrecking even the best design during the process of construction and before the structure is put into service. Under the circumstances it seems in order to call attention to some of these factors which, in themselves, may influence the ultimate design.

Whenever cells are to be driven through a stratum of mud or soft clay, which is unsuitable for cell fill materials because of its low shearing resistance, it is advisable to excavate and remove such objectionable material before driving the cells. Where this is not feasible, the mud inside of the cells must be replaced by sand and gravel by a carefully controlled process of displacement to avoid collapsing the cells due to external pressures.

Sheet-pile cellular walls should be treated as living structures. It is always

TABLE 1.—COMPARISON OF SHEET PILING FOR COF



Plan

Item	Length (ft)	Sheet Piles (Tons)		Earth Fill (Cu Yd)		Estimated Costs, Piles, at $56 Per Ton		Fill	
		Total	Unit	Total	Unit	Total	Unit[a]	Total	Unit[a]
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
1	123.33	341.83	2.772	10,965	88.908	19,142.48	155.21	3,618.45[f]	29.34[f]
2	124.00	340.45	2.746	10,832	87.355	19,065.20	153.75	4,116.16[g]	33.19[g]

[a] Dollars per longitudinal foot of cofferdam. [b] At $10.00 per ton. [c] Maximum compression, in pounds $0.33 per cu yd. [g] At $0.38 per cu yd.

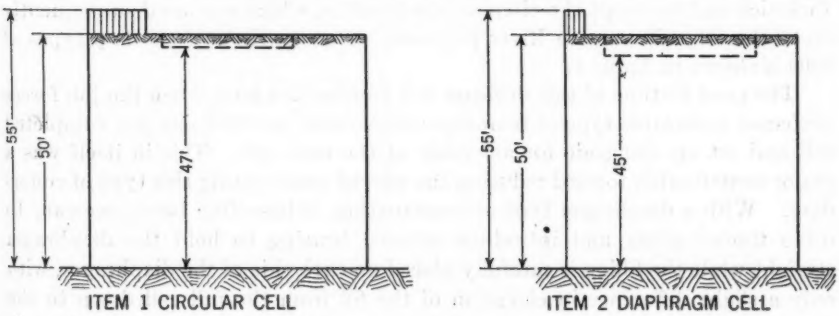
advisable to maintain field observation with respect to deflection, water pressure, and saturation lines, in order to understand all of the forces acting on the structure, and to secure data for future designs.

Some unusual experiences were encountered in driving piles at Madden Dam (in the Isthmus of Panama) through a gravel formation containing many boulders which were not disclosed in the original exploratory borings. This pointed to the advisability of driving a test cell on large projects, ahead of the main undertaking of cofferdam construction; such a cell can be excavated to rock and the conditions of the interlock observed as well as the penetration into the rock. If such a procedure is followed with a careful recording of the type of hammer and energy required to drive the piling, formation through which driving occurs, amount of ground water encountered, and similar data, the cost of such a test cell is quickly recovered in the better procedures that can be adopted for driving and subsequent pulling of the main cofferdams. Such a test cell was employed to advantage at Pickwick Landing Dam and others built subsequently.

Pile driving and construction procedures have a vital bearing on the successful application of a design in which sheet-pile cellular walls are used.

Often the driving of sheet piling is considered an ordinary operation, and the highly specialized techniques are frequently ignored. This results in typical "butchery" of the sheet piling, making it practically useless for re-driving or

FERDAM CELLS; PICKWICK LANDING DAM, TENNESSEE



Sections Through Cofferdam

IN DOLLARS				Maximum Compression[c]	MAXIMUM RESISTING OVERTURNING MOMENT[d]		MAXIMUM FRICTION AT THE BASE[e]		Item
Driving[b]		All Items							
Total	Unit[a]	Total	Unit[a]		Total	Unit	Total	Unit	
(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	
3,418.30	27.72	26,179.23	212.27	11,000	958,746,500	7,771,600	16,281,380	132,015	1
3,404.50	27.46	26,585.86	214.40	11,000	803,589,000	6,480,500	16,084,640	129,715	2

per square foot, when there is no uplift. [d] In foot-pounds. [e] In pounds; coefficient assumed as 0.5. [f] At

salvage. A sheet pile, deflected by a boulder, can curl underground in such a way that, if driving is continued, the entire pile may be dislocated. Furthermore, driving out of the interlock is a possibility which must be controlled properly. Where there is a well-developed understanding of the formation through which piles are to be driven and where there is competent and experienced pile driving supervision with proper recognition of the design objectives and limitations, a cellular wall structure of high structural capacity can be secured.

Pickwick Landing Dam was the first of the large projects on the Tennessee River requiring sheet-pile cellular cofferdams. In view of the fact that a number of important projects had previously been constructed with the diaphragm type of cofferdams, under heads corresponding to those encountered at Pickwick, the diaphragm type was adopted at the outset. This was also in accordance with general advice offered by sheet-pile manufacturers. This plan required the minimum tonnage of steel and presumably the manufacturers considered that they were acting in the best interests of the customer by saving steel. However, in the process of studying the construction procedures and other details more carefully, the writer decided to inspect three cofferdams, then in operation, in the eastern part of the United States, at Gallipolis Dam on the Ohio River Tygart Dam in West Virginia, and on the Fear River in South Carolina. He was impressed with the fact that in each case the circular cell type of cofferdam had been used and certain practical construction advantages were evident. This experience led him to abandon the original plans for Pickwick and to adopt the circular construction, which was used subsequently on all the other Tennessee River projects. A comparison of the two types of cells is shown in Table 1.

The good fortune of this decision was emphasized later when the job forces perfected a portable type of template which could be lifted out of a completed cell and set up alongside for assembly of the next cell. This in itself was a major contribution toward reducing the cost of constructing this type of cofferdam. With a diaphragm type of construction, it has often been necessary to drive timber piling and introduce internal bracing to hold the diaphragm straight while the filling is carefully placed on each side of the diaphragm, with only a slight difference in elevation of the fill from the full cell down to the empty cell, ranging in steps over eight or ten cells. With circular cells, it is feasible to fill each cell to the top. Furthermore, in the event of damage to, or failure of, a circular cell, the trouble remains localized and can be repaired more readily. The use of the diaphragm type of cellular wall is justified in special cases where there is no problem of filling the cells, or holding the empty cells in place until they can be filled. The simplest way to fill circular cells is by means of a suction dredge and here again, if there is an objectionable percentage of fines in the materials, the suction dredge tends to churn up the material and throw the fines into suspension so they can be drained off, as the heavier and sounder materials settle into the cells.

It is of interest to note in Table 1 that, for cofferdams higher than 35 ft, there is little difference in the sheet-pile tonnage in the circular cell or the diaphragm

type cell, but there are great savings in the cost of constructing the circular type of cofferdam where it is possible to employ a cheaper method of filling the cells.

No doubt the art of design in the type of structure discussed herein would be advanced considerably if a precise record of stress measurements could be made on a full-sized structure to determine actual stress distribution in the steel cell, as well as in the fill and in the supporting foundation. It would appear that the promotion of such a research project deserves further consideration.

Bibliography.—

(34) **"Cofferdam Design for Kentucky Dam,"** by A. F. Hedman, *Engineering News-Record*, Vol. 128, 1942, pp. 30–35.

(35) **"Big Rigs Drive Cofferdam Piles,"** by J. W. Peerson, *ibid.*, pp. 194–199.

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DISCUSSIONS

ANALYSIS OF RIGID FRAMES BY SUPERPOSITION

Discussion

BY DAVID M. WILSON, M. AM. SOC. C. E.

DAVID M. WILSON,[16] M. AM. SOC. C. E.[16a]—Several of the discussions have concerned methods of balancing moments at the joints in step 3 of the paper. Messrs. Stewart and McGee prefer the traverse method (17)[16b] for this purpose, whereas Messrs. Polivka and Eremin use the graphical method of fixed points. Although these methods have definite limitations (17*a*) not possessed by the general slope-deflection and moment distribution methods, they will be found useful in the analysis of continuous beams and one-story frames. In this connection, a special technique in the use of slope deflection will be presented subsequently which accomplishes the same purpose as the traverse method.

Mr. Fischer presents some short cuts in the application of moment distribution to several of the simple examples of the paper. These are useful to the experienced designer in simplifying distribution calculations for many problems where they apply. In the paper, short cuts were intentionally avoided in order that the usefulness of superposition might be demonstrated without giving undue attention to distribution procedures. Furthermore, the slope-deflection and moment distribution methods, as used in the paper, are well known and perfectly general in their application to the analyses of all types of rigid frames.

Mr. Merritt questions the validity of conclusion 4. The writer agrees with him that nearly all rigid-frame problems can be solved most expeditiously by moment distribution. However, in certain cases, slope-deflection procedures will furnish a more rapid convergence than moment distribution, as Mr. Goldberg states in his discussion. A comprehensive treatment of the limits of validity of the principle of superposition has been presented by J. A. Van den

NOTE.—This paper by David M. Wilson was published in February, 1944, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: May, 1944, by F. S. Merritt, Ralph W. Stewart, and John E. Goldberg; June, 1944, by A. W. Fischer, Leon Blog, Arthur B. McGee, and Jaroslav Polivka; September, 1944, by Odd Albert, and W. C. Spiker; and November, 1944, by David B. Hall, A. A. Eremin, and Victor R. Bergman.

[16] Prof., Civ. Eng., Univ. of Southern California, Los Angeles, Calif.

[16a] Received by the Secretary February 6, 1945.

[16b] Numerals in parentheses, thus: (17), refer to corresponding items in the Bibliography (see Appendix of the paper), and at the end of discussion in this issue.

Broek, M. Am. Soc. C. E. (11*a*). The treatment of moving loads, suggested also by Mr. Merritt, will be given attention subsequently.

Mr. Albert suggests that the method of procedure is improved by determining the magnitude of the horizontal force required to prevent sidesway in Fig. 27. This method is presented in textbooks (3*b*) and elsewhere but, in the writer's opinion, is indirect and requires more computations than the method of the paper. Of course, certain forces must be assumed to act during the several deformation conditions of any frame; but it is not necessary to determine the magnitudes of these forces since they cancel each other when the resultant deformation effects are considered.

Mr. Hall presents an interesting and instructive discussion of the determination of the relative R-values for members of a quadrangle by reference to the instantaneous center of rotation. For the same purpose, Messrs. McGee and Bergman use a convenient semi-graphical method which is familiar to most structural analysts (3*c*). The question by Mr. Bergman concerning the analysis of frames having members of varying cross section is answered by Messrs. McGee and Blog and is given further consideration in this closure.

TABLE 4.—DISTRIBUTION OF APPLIED MOMENT OF 1,000 FT-LB AT JOINT B, FIG. 2 (CLOCKWISE)

Description	Joint and Member (See Fig. 2(*a*))									
	A	B		C			D	E		F
	AB	BA	BC	CB	CD	CE	DC	EC	EF	FE
Distribution Factor.....		0.333	0.667	0.40	0.20	0.40		0.727	0.273	
Moment distributed....		333	667							
Moment carried over...				333						
Moment distributed....				−133	−67	−133				
Moment carried over...			−67					−67		
Moment distributed....		22	45					49	18	
Moment carried over...				22		24				
Moment distributed....				−18	−10	−18				
Moment carried over...			−9					−9		
Moment distributed....		3	6					7	2	
Final moment.....	179	358	642	204	−77	−127	−38.5	−20	20	0

To clarify the general questions raised and problems suggested in the discussions, the following comments will deal with (*a*) the treatment of moving loads, and (*b*) the presentation of the general slope-deflection equation for members having varying moments of inertia in a form convenient for use in either the slope-deflection or moment distribution method of analysis. The part concerning moving loads will include the general solution by moment distribution and a special solution by slope deflection (applicable to continuous beams and one-story frames) which accomplishes the same purpose as the

traverse method (17). The part dealing with varying moment of inertia will include a special solution by slope deflection, similar to that used for frames having members of constant section, of the frame analyzed by Mr. McGee.

Moving Loads.—In the solution of problems involving moving loads, it is recommended that the effect of an applied moment at each joint on all members of the frame be determined at the outset, either by moment distribution or by slope deflection. The resultant effects of different combinations of loads, both with and without sidesway, can then be determined readily by proportion and superposition without additional calculations involving the balancing of moments at the joints.

General Solution by Moment Distribution.—Example 2 of the paper has been selected to illustrate the foregoing procedure, using moment distribution. Table 4(*a*) shows the distribution of an applied moment of 1,000 ft-lb (assumed) at joint B (Fig. 2). Similarly, effects of applied moments of 1,000 ft-lb each at joints C and E are determined, and a summary of the results obtained is given in Table 5. From the values in Table 5 (called flexure factors by Mr. Stewart (17)), the distribution of a required balancing moment at any joint can be determined by simple proportion. The balancing moment may be either a numerical value, or a function of unknown displacements, as explained in the paper. After all joints are balanced, steps 4 and 5 give a solution for each critical position of the loads. If influence lines are desired, several solutions for varying positions of a unit load must be made.

TABLE 5.—EFFECTS OF A MOMENT OF 1,000 FT-LB APPLIED AT JOINTS B, C, AND E, FIG. 2. (CLOCKWISE)

Joint	MEMBER (SEE FIG. 2(*a*))									
	AB	BA	BC	CB	CD	CE	DC	EC	EF	FE
B	179	358	642	204	−77	−127	−38.5	−20	20	0
C	−38	−76	76	388	231	381	115	63	−63	0
E	13.5	27	−27	−141	−83	224	−41.5	704	296	0

Special Solution by Slope Deflection.—A special solution by slope deflection of continuous beams and one-story frames, which accomplishes the same purpose as the traverse method (17), is very useful. Example 2 of the paper also has been chosen to demonstrate this application of slope deflection.

For the purpose of determining the effects of an applied moment at any joint, the slope-deflection equation may be written conveniently in the following form, dropping the constant factor $2E$:

$$M\ (\text{near}) = K\ [2\ \theta\ (\text{near}) + \theta\ (\text{far})] \ldots\ldots\ldots\ldots\ldots (88)$$

In Fig. 33(*a*), it is first assumed that the rotation angle at joint F is equal to unity. By applying Eq. 88 successively to the several members of the frame, the following simple calculations are made. First assume $\theta_F = 1$; then:

$M_{FE} = 1\,(2\,\theta_F + \theta_E) = 0; \quad \theta_E = -2$
$M_{EF} = 1\,(2\,\theta_E + \theta_F) = -4 + 1 = -3$

$M_{EC} = 2\,(2\,\theta_E + \theta_C) = 3; \quad \theta_C = 1.5 + 4 = 5.5$
$M_{CD} = 1\,(2\,\theta_C + 0) = 11$
$M_{CE} = 2\,(2\,\theta_C + \theta_E) = 2\,(11 - 2) = 18$

$M_{CB} = 2\,(2\,\theta_C + \theta_B) = 2\,(11 + \theta_B) = -29; \quad \theta_B = -14.5 - 11 = -25.5$
$M_{BC} = 2\,(2\,\theta_B + \theta_C) = 2\,(-51 + 5.5) = -91$

$M_{BA} = 1\,(2\,\theta_B + 0) = -51$
$M_{AB} = 1\,(0 + \theta_B) = -25.5$

The moment values thus obtained are shown at the ends of the members in Fig. 33(*a*). It is obvious that an applied moment of −142 ft-lb at joint B will induce these moments throughout the frame.

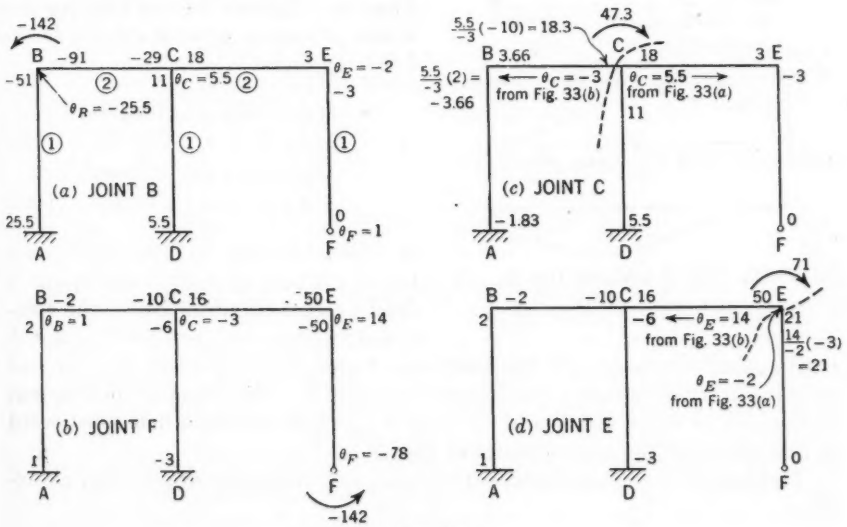


FIG. 33

Similarly, if it is assumed that the rotation angle at joint B is equal to unity, the moment values shown in Fig. 33(*b*) are obtained. It will be noted that a moment of −142 ft-lb at joint F is required to produce unit rotation at joint B, whereas (see Fig. 33(*a*)) a moment of −142 ft-lb at joint B produces unit rotation at F. This is in agreement with Maxwell's law of reciprocity of displacement and constitutes a valuable check on the calculations.

From the moment values shown in Figs. 33(*a*) and 33(*b*), the effects of an applied moment at either joint C or joint E can be determined readily as shown in Figs. 33(*c*) and 33(*d*).

Varying Moment of Inertia.—In the paper, it was stated that the method of procedure therein described is general and can be applied with little additional

labor to the analysis of frames having members of varying section after fixed-end moments, carry-over factors, and stiffness values have been evaluated. Mr. McGee has shown in an excellent manner the application of the method of procedure to a two-bay, saw-toothed structure with tapering members. Inasmuch as he used the traverse method in balancing moments at the joints, it is considered desirable to present the general slope-deflection equation for members having varying moments of inertia in a form convenient for use in either the slope-deflection or moment distribution method of analysis. Reference to Fig. 34 will assist in making clear the meaning of the symbols.

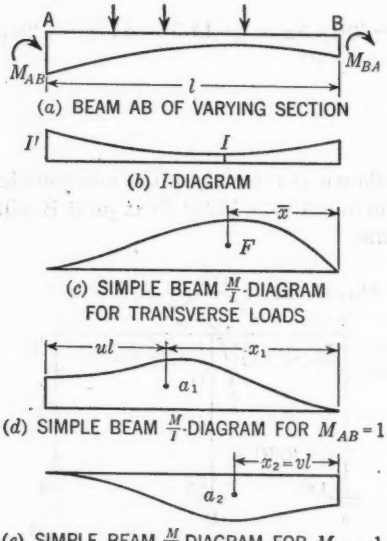


FIG. 34.—TYPICAL $\frac{M}{I}$-DIAGRAMS

General Slope-Deflection Equations.—The general slope-deflection equations for members having varying moments of inertia may be written in the following form:

$$M_{AB} = S_A(\theta_A + c_A\theta_B) - S_A(1 + c_A)R + C_{AB} \quad (89a)$$

$$M_{BA} = S_B(\theta_B + c_B\theta_A) - S_B(1 + c_B)R + C_{BA} \quad (89b)$$

in which (see Fig. 34): S_A is the rotational stiffness at joint A with joint B fixed against rotation; S_B is the rotational stiffness at joint B with joint A fixed against rotation; c_A is the carry-over factor from joint A to joint B; and c_B is the carry-over factor from joint B to joint A. For members of constant section, $S_A = S_B = 4EK$ and $c_A = c_B = \frac{1}{2}$, which values, when substituted in Eqs. 89, yield the familiar form of Eqs. 2.

Stiffnesses—Far End Fixed.—The rotational stiffnesses are defined as follows:

$$S_A = \frac{E(l - x_2)}{a_1(x_1 - x_2)} = \frac{EI}{l}\frac{1 - v}{p(1 - u - v)} = \frac{EI}{l}\alpha \quad (90a)$$

$$S_B = \frac{Ex_1}{a_2(x_1 - x_2)} = \frac{EI}{l}\frac{1 - u}{q(1 - u - v)} = \frac{EI}{l}\beta \quad (90b)$$

in which (21): $a_1 = \frac{pl}{I}$ = area of a simple-beam M-diagram for $M_{AB} = 1$ (assumed a positive quantity); $a_2 = \frac{ql}{I}$ = the area of a simple-beam M-diagram for $M_{BA} = 1$ (assumed a positive quantity); $x_1 = l - ul$ = the distance from the centroid of the $\frac{M}{I}$-diagram for M_{AB} to end B of the member; $x_2 = vl$ = the

distance from the centroid of the $\frac{M}{I}$-diagram for M_{BA} to end B of the member; u and v are ratios introduced by Walter Ruppel, Assoc. M. Am. Soc. C. E. (21); and α and β are stiffness coefficients defined as follows:

$$\alpha = \frac{1 - v}{p\,(1 - u - v)} \quad \ldots\ldots (91a)$$

and

$$\beta = \frac{1 - u}{q\,(1 - u - v)} \quad \ldots\ldots (91b)$$

For members having constant section, $\alpha = \beta = 4$.

Carry-Over Factors.—The carry-over factors in Eqs. 89 are defined as follows:

$$c_A = \frac{x_2}{l - x_2} = \frac{v}{1 - v} \quad \ldots\ldots (92a)$$

$$c_B = \frac{l - x_1}{x_1} = \frac{u}{1 - u} \quad \ldots\ldots (92b)$$

Fixed-End Moments.—The fixed-end moments caused by transverse loads on the member AB are defined as follows:

$$C_{AB} = -\frac{F\,(\bar{x} - x_2)}{a_1\,(x_1 - x_2)} = -\frac{F\,(\bar{x} - v\,l)\,I}{p\,l^2\,(1 - u - v)} \quad \ldots\ldots (93a)$$

$$C_{BA} = +\frac{F\,(x_1 - \bar{x})}{a_2\,(x_1 - x_2)} = +\frac{F\,(l - u\,l - \bar{x})\,I}{q\,l^2\,(1 - u - v)} \quad \ldots\ldots (93b)$$

The fixed-end moments induced by angular displacement, R, are

$$M'_{AB} = -S_A\,(1 + c_A)\,R = -S_A\,(1 + c_A)\,\frac{\Delta}{l} \quad \ldots\ldots (94a)$$

$$M'_{BA} = -S_B\,(1 + c_B)\,R = -S_B\,(1 + c_B)\,\frac{\Delta}{l} \quad \ldots\ldots (94b)$$

It should be noted that I in the foregoing formulas for stiffnesses and fixed-end moments is the minimum moment of inertia of the member. A valuable check of the calculated values of stiffnesses and carry-over factors is given by the following relation:

$$S_A\,c_A = S_B\,c_B \quad \ldots\ldots (95)$$

Special Slope-Deflection Equation.—If one end (say, end B) of the member is free to rotate so that $M_{BA} = 0$, the following special slope-deflection equation can be derived from Eqs. 89:

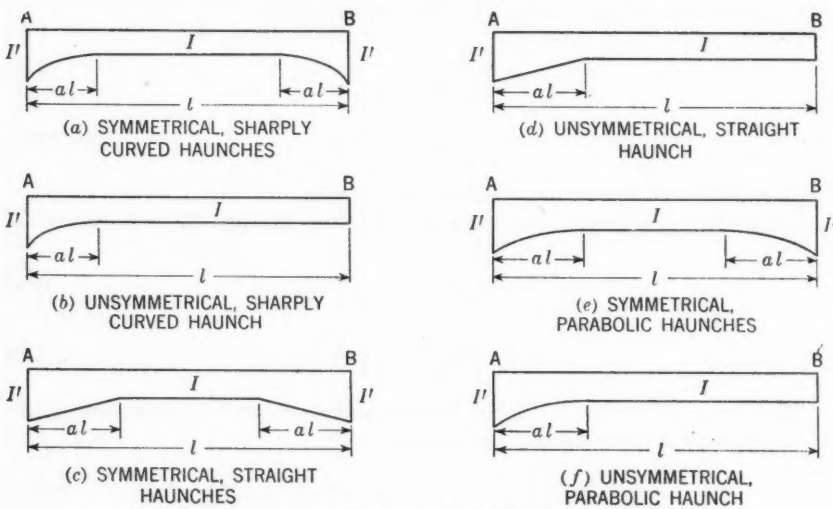
$$M_{AB} = S_A\,(1 - c_A\,c_B)\,\theta_A - S_A\,(1 - c_A\,c_B)\,R + H_{AB} \quad \ldots\ldots (96)$$

in which H_{AB} = the fixed-end moment at end A caused by transverse loads on the member, with end B hinged—expressed as,

$$H_{AB} = C_{AB} - c_B\,C_{BA} \quad \ldots\ldots (97)$$

From Eq. 96, it is obvious that the rotational stiffness at end A of the member is $(1 - c_A\,c_B)$ times the rotational stiffness at end A when end B is

TABLE 6.—Beam Coefficients for Members of Varying Moment of Inertia



(*a*) SYMMETRICAL, SHARPLY CURVED HAUNCHES

(*b*) UNSYMMETRICAL, SHARPLY CURVED HAUNCH

(*c*) SYMMETRICAL, STRAIGHT HAUNCHES

(*d*) UNSYMMETRICAL, STRAIGHT HAUNCH

(*e*) SYMMETRICAL, PARABOLIC HAUNCHES

(*f*) UNSYMMETRICAL, PARABOLIC HAUNCH

$\left(S_A = \alpha \frac{EI}{l}; \text{ and } S_b = \beta \frac{EI}{l}\right)$

Quantity	Values of $b = I/I'$												
	1.00	0.60	0.30	0.20	0.15	0.12	0.10	0.08	0.06	0.05	0.04	0.03	0.02
(*a*) Symmetrical, Sharply-Curved Haunches													
$a = 0.50$—													
$c_A = c_B$	0.500	0.548	0.607	0.636	0.653	0.664	0.673	0.676	0.686	0.692	0.694	0.701	0.703
$\alpha = \beta$	4.00	5.54	7.85	9.17	10.00	10.63	11.10	11.50	12.01	12.33	12.60	13.00	13.23
$1 - c_A c_B$	0.75	0.700	0.631	0.595	0.574	0.560	0.547	0.542	0.529	0.521	0.518	0.509	0.506
$a = 0.40$—													
$c_A = c_B$	0.500	0.553	0.612	0.639	0.653	0.661	0.669	0.674	0.684	0.687	0.688	0.695	0.698
$\alpha = \beta$	4.00	5.32	7.18	8.16	8.73	9.12	9.45	9.75	10.12	10.28	10.43	10.70	10.88
$1 - c_A c_B$	0.75	0.694	0.625	0.592	0.574	0.563	0.553	0.545	0.533	0.528	0.526	0.517	0.513
$a = 0.35$—													
$c_A = c_B$	0.500	0.553	0.610	0.650	0.647	0.656	0.661	0.667	0.675	0.676	0.680	0.684	0.686
$\alpha = \beta$	4.00	5.19	6.79	7.59	8.08	8.38	8.63	8.85	9.19	9.30	9.43	9.55	9.70
$1 - c_A c_B$	0.75	0.694	0.628	0.578	0.581	0.569	0.563	0.556	0.545	0.542	0.537	0.533	0.528
$a = 0.30$—													
$c_A = c_B$	0.500	0.553	0.605	0.626	0.636	0.644	0.650	0.655	0.658	0.663	0.667	0.667	0.669
$\alpha = \beta$	4.00	5.07	6.41	7.03	7.38	7.65	7.84	8.02	8.15	8.30	8.43	8.45	8.57
$1 - c_A c_B$	0.75	0.694	0.634	0.608	0.595	0.585	0.578	0.570	0.567	0.560	0.556	0.556	0.553
$a = 0.25$—													
$c_A = c_B$	0.500	0.548	0.595	0.612	0.623	0.629	0.634	0.636	0.642	0.644	0.644	0.647	0.650
$\alpha = \beta$	4.00	5.00	5.98	6.46	6.74	6.90	7.04	7.15	7.31	7.38	7.40	7.48	7.56
$1 - c_A c_B$	0.75	0.700	0.646	0.625	0.612	0.604	0.599	0.595	0.588	0.585	0.585	0.581	0.578
$a = 0.20$—													
$c_A = c_B$	0.500	0.543	0.582	0.597	0.605	0.610	0.612	0.618	0.620	0.623	0.623	0.626	0.626
$\alpha = \beta$	4.00	4.76	5.57	5.92	6.10	6.23	6.30	6.41	6.49	6.55	6.57	6.64	6.65
$1 - c_A c_B$	0.75	0.705	0.661	0.643	0.634	0.628	0.625	0.618	0.615	0.612	0.612	0.608	0.608
$a = 0.15$—													
$c_A = c_B$	0.500	0.536	0.552	0.577	0.584	0.587	0.590	0.592	0.594	0.594	0.597	0.597	0.600
$\alpha = \beta$	4.00	4.39	5.16	5.37	5.53	5.59	5.64	5.69	5.74	5.75	5.81	5.82	5.86
$1 - c_A c_B$	0.75	0.712	0.695	0.667	0.658	0.655	0.652	0.649	0.647	0.647	0.644	0.643	0.640

TABLE 6.—(*Continued*)

Quantity	Values of $b = \dot{I}/I'$												
	1.00	0.60	0.30	0.20	0.15	0.12	0.10	0.08	0.06	0.05	0.04	0.03	0.02
(*b*) Unsymmetrical, Sharply-Curved Haunch													
$a=1.00$—													
c_A	0.500	0.445	0.394	0.375	0.364	0.358	0.355	0.351	0.346	0.344	0.342	0.340	0.337
c_B	0.500	0.572	0.684	0.750	0.792	0.825	0.842	0.874	0.895	0.914	0.929	0.940	0.960
α	4.00	5.74	8.64	10.47	11.60	12.53	13.17	13.78	14.70	15.13	15.67	16.15	16.75
β	4.00	4.48	4.98	5.22	5.35	5.46	5.53	5.53	5.70	5.72	5.77	5.84	5.87
$1-c_A c_B$	0.75	0.746	0.730	0.718	0.712	0.704	0.701	0.693	0.690	0.686	0.682	0.680	0.676
$a=0.50$—													
c_A	0.500	0.468	0.445	0.436	0.432	0.429	0.428	0.426	0.424	0.424	0.422	0.422	0.420
c_B	0.500	0.587	0.692	0.739	0.765	0.785	0.794	0.810	0.823	0.830	0.840	0.844	0.851
α	4.00	5.26	6.91	7.72	8.19	8.51	8.74	9.00	9.20	9.38	9.49	9.58	9.74
β	4.00	4.19	4.43	4.54	4.63	4.65	4.70	4.72	4.74	4.79	4.78	4.80	4.82
$1-c_A c_B$	0.75	0.725	0.692	0.678	0.669	0.663	0.660	0.654	0.651	0.649	0.645	0.644	0.642
$a=0.40$—													
c_A	0.500	0.477	0.460	0.455	0.451	0.450	0.449	0.446	0.446	0.445	0.445	0.445	0.445
c_B	0.500	0.580	0.670	0.705	0.728	0.739	0.750	0.761	0.770	0.775	0.778	0.785	0.792
α	4.00	5.07	6.36	6.96	7.30	7.49	7.69	7.85	8.01	8.08	8.17	8.27	8.40
β	4.00	4.16	4.38	4.49	4.52	4.55	4.60	4.61	4.65	4.63	4.67	4.70	4.71
$1-c_A c_B$	0.75	0.723	0.692	0.679	0.672	0.668	0.664	0.660	0.656	0.655	0.654	0.650	0.647
$a=0.35$—													
c_A	0.500	0.481	0.468	0.464	0.462	0.460	0.460	0.457	0.457	0.455	0.455	0.455	0.455
c_B	0.500	0.575	0.654	0.686	0.702	0.716	0.724	0.730	0.740	0.746	0.748	0.754	0.759
α	4.00	4.97	6.08	6.57	6.83	7.02	7.15	7.26	7.42	7.49	7.54	7.65	7.72
β	4.00	4.16	4.35	4.44	4.49	4.50	4.54	4.55	4.58	4.58	4.60	4.62	4.62
$1-c_A c_B$	0.75	0.723	0.694	0.682	0.676	0.671	0.667	0.666	0.662	0.660	0.660	0.657	0.655
$a=0.30$—													
c_A	0.500	0.486	0.475	0.473	0.470	0.468	0.468	0.468	0.466	0.466	0.466	0.466	0.466
c_B	0.500	0.567	0.636	0.664	0.680	0.689	0.695	0.701	0.710	0.714	0.716	0.717	0.720
α	4.00	4.85	5.79	6.17	6.43	6.55	6.65	6.75	6.85	6.88	6.95	7.00	7.05
β	4.00	4.15	4.32	4.40	4.44	4.45	4.47	4.49	4.50	4.50	4.52	4.55	4.55
$1-c_A c_B$	0.75	0.724	0.698	0.694	0.680	0.678	0.675	0.672	0.669	0.667	0.666	0.665	0.664
$a=0.25$—													
c_A	0.500	0.490	0.481	0.478	0.479	0.476	0.476	0.476	0.476	0.475	0.475	0.475	0.475
c_B	0.500	0.560	0.618	0.639	0.652	0.659	0.663	0.670	0.674	0.676	0.680	0.685	0.687
α	4.00	4.74	5.49	5.80	5.97	6.06	6.14	6.20	6.30	6.33	6.36	6.40	6.45
β	4.00	4.14	4.28	4.33	4.38	4.38	4.41	4.42	4.45	4.44	4.44	4.45	4.46
$1-c_A c_B$	0.75	0.726	0.702	0.694	0.688	0.686	0.684	0.680	0.679	0.679	0.677	0.675	0.674
$a=0.20$—													
c_A	0.500	0.492	0.488	0.486	0.486	0.486	0.486	0.484	0.484	0.485	0.484	0.485	0.485
c_B	0.500	0.550	0.597	0.615	0.622	0.629	0.634	0.635	0.640	0.642	0.645	0.646	0.650
α	4.00	4.60	5.19	5.42	5.54	5.60	5.66	5.71	5.75	5.78	5.83	5.85	5.88
β	4.00	4.11	4.24	4.29	4.32	4.33	4.35	4.34	4.35	4.35	4.37	4.39	4.39
$1-c_A c_B$	0.75	0.729	0.708	0.700	0.698	0.694	0.692	0.692	0.691	0.688	0.688	0.686	0.685
$a=0.15$—													
c_A	0.500	0.497	0.492	0.492	0.493	0.492	0.490	0.490	0.490	0.490	0.490	0.490	0.490
c_B	0.500	0.541	0.575	0.587	0.594	0.596	0.600	0.603	0.605	0.607	0.607	0.610	0.610
α	4.00	4.46	4.88	5.04	5.13	5.17	5.20	5.25	5.28	5.30	5.30	5.34	5.35
β	4.00	4.10	4.19	4.22	4.25	4.25	4.25	4.25	4.26	4.28	4.28	4.29	4.29
$1-c_A c_B$	0.75	0.731	0.717	0.711	0.707	0.706	0.706	0.704	0.703	0.702	0.702	0.702	0.701
(*c*) Symmetrical Straight Haunches													
$a=0.50$—													
$c_A=c_B$	0.500	0.547	0.613	0.650	0.675	0.695	0.713	0.730	0.754	0.766	0.782	0.805	0.835
$\alpha=\beta$	4.00	5.69	9.25	12.32	15.15	17.80	20.40	24.05	29.10	34.00	40.00	50.25	70.40
$1-c_A c_B$	0.75	0.700	0.624	0.578	0.545	0.517	0.492	0.467	0.432	0.413	0.388	0.352	0.304
$a=0.40$—													
$c_A=c_B$	0.500	0.557	0.631	0.673	0.683	0.724	0.739	0.759	0.785	0.797	0.815	0.835	0.859
$\alpha=\beta$	4.00	5.50	8.36	10.68	12.70	14.60	16.25	18.50	22.30	24.70	28.10	33.30	41.80
$1-c_A c_B$	0.75	0.690	0.602	0.547	0.534	0.476	0.455	0.425	0.384	0.364	0.336	0.303	0.263
$a=0.35$—													
$c_A=c_B$	0.500	0.557	0.631	0.673	0.683	0.721	0.735	0.754	0.776	0.789	0.805	0.818	0.843
$\alpha=\beta$	4.00	5.35	7.84	9.79	11.43	12.87	14.10	15.79	18.23	19.90	22.30	24.90	30.10
$1-c_A c_B$	0.75	0.690	0.602	0.547	0.534	0.480	0.460	0.432	0.397	0.377	0.351	0.331	0.290
$a=0.30$—													
$c_A=c_B$	0.500	0.555	0.626	0.667	0.689	0.710	0.720	0.739	0.757	0.766	0.779	0.795	0.811
$\alpha=\beta$	4.00	5.19	7.26	8.85	10.00	11.10	11.87	13.13	14.61	15.52	16.88	18.70	21.05
$1-c_A c_B$	0.75	0.692	0.608	0.556	0.525	0.496	0.481	0.455	0.426	0.413	0.394	0.368	0.342

TABLE 6.—(*Continued*)

Quantity	Values of $b = I/I'$												
	1.00	0.60	0.30	0.20	0.15	0.12	0.10	0.08	0.06	0.05	0.04	0.03	0.02
(c) (*Continued*)													
$a=0.25$—													
$c_A=c_B$	0.500	0.553	0.618	0.650	0.672	0.687	0.700	0.712	0.730	0.739	0.748	0.760	0.774
$\alpha=\beta$	4.00	5.03	6.71	7.81	8.67	9.32	9.96	10.62	11.65	12.25	12.94	13.88	15.02
$1-c_A c_B$	0.75	0.694	0.618	0.578	0.548	0.528	0.510	0.493	0.466	0.454	0.441	0.422	0.401
$a=0.20$—													
$c_A=c_B$	0.500	0.547	0.603	0.631	0.647	0.660	0.670	0.680	0.692	0.701	0.710	0.717	0.730
$\alpha=\beta$	4.00	4.86	6.10	6.92	7.45	7.90	8.22	8.65	9.17	9.55	9.94	10.40	11.10
$1-c_A c_B$	0.75	0.700	0.636	0.602	0.581	0.564	0.551	0.537	0.521	0.508	0.496	0.485	0.466
$a=0.15$—													
$c_A=c_B$	0.500	0.538	0.581	0.605	0.618	0.626	0.634	0.642	0.650	0.655	0.660	0.669	0.677
$\alpha=\beta$	4.00	4.65	5.51	6.02	6.37	6.60	6.81	7.05	7.31	7.48	7.68	7.95	8.26
$1-c_A c_B$	0.75	0.710	0.662	0.634	0.618	0.608	0.599	0.588	0.578	0.570	0.564	0.552	0.541
(d) Unsymmetrical, Straight Haunch													
$a=1.00$—													
c_A	0.500	0.441	0.367	0.333	0.309	0.292	0.279	0.264	0.244	0.232	0.218	0.202	0.181
c_B	0.500	0.567	0.674	0.746	0.794	0.844	0.880	0.930	1.00	1.04	1.090	1.180	1.307
α	4.00	5.86	9.95	13.56	16.85	20.05	23.00	27.40	34.30	39.60	46.70	59.30	81.50
β	4.00	4.55	5.44	6.07	6.52	6.94	7.30	7.74	8.38	8.80	9.37	10.10	11.35
$1-c_A c_B$	0.75	0.749	0.752	0.752	0.755	0.754	0.755	0.754	0.756	0.758	0.762	0.763	0.764
$a=0.50$—													
c_A	0.500	0.464	0.422	0.404	0.393	0.383	0.377	0.370	0.363	0.357	0.351	0.345	0.337
c_B	0.500	0.595	0.739	0.835	0.905	0.960	1.005	1.060	1.133	1.175	1.230	1.290	1.380
α	4.00	5.40	7.88	9.75	11.27	12.50	13.63	15.09	17.02	18.25	19.85	22.00	25.10
β	4.00	4.21	4.50	4.74	4.89	5.00	5.12	5.26	5.45	5.54	5.69	5.86	6.11
$1-c_A c_B$	0.75	0.724	0.688	0.663	0.644	0.632	0.621	0.607	0.589	0.580	0.569	0.555	0.535
$a=0.40$—													
c_A	0.500	0.473	0.445	0.431	0.423	0.416	0.412	0.406	0.401	0.396	0.393	0.389	0.384
c_B	0.500	0.587	0.715	0.790	0.845	0.888	0.918	0.955	1.010	1.035	0.071	1.115	1.162
α	4.00	5.19	7.19	8.48	9.55	10.40	11.07	11.90	13.03	13.80	14.62	15.73	17.20
β	4.00	4.18	4.47	4.63	4.79	4.89	4.97	5.08	5.20	5.29	5.36	5.50	5.65
$1-c_A c_B$	0.75	0.722	0.681	0.659	0.643	0.630	0.622	0.612	0.596	0.590	0.579	0.565	0.554
$a=0.35$—													
c_A	0.500	0.479	0.455	0.444	0.439	0.433	0.429	0.424	0.420	0.419	0.414	0.410	0.405
c_B	0.500	0.581	0.695	0.762	0.810	0.840	0.869	0.900	0.940	0.965	0.990	1.020	1.062
α	4.00	5.08	6.77	7.90	8.73	9.30	9.86	10.50	11.33	11.90	12.50	13.15	14.20
β	4.00	4.19	4.44	4.63	4.73	4.79	4.86	4.96	5.08	5.15	5.21	5.30	5.40
$1-c_A c_B$	0.75	0.722	0.684	0.662	0.645	0.636	0.628	0.619	0.604	0.595	0.590	0.582	0.570
$a=0.30$—													
c_A	0.500	0.484	0.466	0.457	0.454	0.449	0.445	0.444	0.439	0.436	0.434	0.430	0.426
c_B	0.500	0.575	0.675	0.730	0.770	0.794	0.819	0.844	0.875	0.892	0.913	0.933	0.970
α	4.00	4.95	6.40	7.26	7.94	8.39	8.78	9.23	9.81	10.17	10.60	11.08	11.80
β	4.00	4.17	4.41	4.55	4.66	4.72	4.85	4.85	4.92	4.97	5.05	5.11	5.20
$1-c_A c_B$	0.75	0.722	0.686	0.666	0.651	0.644	0.636	0.626	0.616	0.610	0.604	0.599	0.586
$a=0.25$—													
c_A	0.500	0.489	0.475	0.468	0.466	0.462	0.460	0.457	0.454	0.454	0.452	0.449	0.446
c_B	0.500	0.566	0.649	0.695	0.724	0.746	0.762	0.780	0.807	0.818	0.834	0.852	0.875
α	4.00	4.83	5.96	6.64	7.11	7.42	7.72	8.05	8.44	8.67	8.96	9.30	9.74
β	4.00	4.15	4.36	4.46	4.56	4.61	4.66	4.71	4.75	4.81	4.85	4.91	4.96
$1-c_A c_B$	0.75	0.723	0.692	0.675	0.662	0.655	0.650	0.643	0.634	0.629	0.622	0.618	0.609
$a=0.20$—													
c_A	0.500	0.492	0.485	0.479	0.476	0.475	0.473	0.473	0.468	0.468	0.466	0.465	0.466
c_B	0.500	0.555	0.622	0.659	0.681	0.700	0.710	0.724	0.740	0.748	0.761	0.775	0.790
α	4.00	4.67	5.55	6.05	6.36	6.60	6.78	7.02	7.27	7.42	7.60	7.84	8.09
β	4.00	4.12	4.31	4.39	4.45	4.50	4.53	4.58	4.60	4.64	4.65	4.70	4.74
$1-c_A c_B$	0.75	0.726	0.698	0.685	0.675	0.668	0.664	0.658	0.654	0.650	0.645	0.640	0.631
$a=0.15$—													
c_A	0.500	0.495	0.490	0.488	0.486	0.484	0.485	0.484	0.481	0.481	0.481	0.479	0.479
c_B	0.500	0.544	0.595	0.620	0.636	0.646	0.655	0.663	0.674	0.685	0.689	0.699	0.705
α	4.00	4.50	5.15	5.49	5.69	5.84	5.94	6.08	6.23	6.35	6.45	6.58	6.73
β	4.00	4.10	4.24	4.31	4.35	4.38	4.40	4.43	4.45	4.47	4.50	4.51	4.55
$1-c_A c_B$	0.75	0.731	0.709	0.698	0.690	0.687	0.682	0.680	0.675	0.670	0.668	0.666	0.662
(e) Symmetrical, Parabolic Haunches													
$a=0.50$—													
$c_A=c_B$	0.500	0.550	0.618	0.652	0.680	0.698	0.713	0.730	0.750	0.762	0.779	0.798	0.821
$\alpha=\beta$	4.00	5.25	7.59	9.35	10.94	12.25	13.50	15.24	17.70	19.40	22.00	25.60	32.30
$1-c_A c_B$	0.75	0.697	0.618	0.575	0.537	0.512	0.492	0.467	0.437	0.419	0.394	0.364	0.325

TABLE 6.—(*Continued*)

Quantity	Values of $b = I/I'$												
	1.00	0.60	0.30	0.20	0.15	0.12	0.10	0.08	0.06	0.05	0.04	0.03	0.02
(*e*) (*Continued*)													
$a=0.40$—													
$c_A=c_B$	0.500	0.550	0.612	0.647	0.673	0.689	0.703	0.718	0.739	0.748	0.762	0.779	0.798
$\alpha=\beta$	4.00	5.06	6.86	8.18	9.30	10.20	11.03	12.03	13.60	14.50	15.90	17.75	20.50
$1-c_A c_B$	0.75	0.697	0.625	0.582	0.548	0.526	0.505	0.485	0.455	0.460	0.419	0.394	0.364
$a=0.35$—													
$c_A=c_B$	0.500	0.548	0.607	0.639	0.664	0.676	0.689	0.703	0.720	0.732	0.743	0.758	0.779
$\alpha=\beta$	4.00	4.95	6.50	7.58	8.50	9.20	9.75	10.53	11.60	12.40	13.13	14.40	16.54
$1-c_A c_B$	0.75	0.700	0.631	0.592	0.560	0.542	0.525	0.506	0.482	0.464	0.448	0.425	0.394
$a=0.30$—													
$c_A=c_B$	0.500	0.545	0.600	0.629	0.646	0.660	0.675	0.687	0.701	0.710	0.720	0.732	0.747
$\alpha=\beta$	4.00	4.84	6.14	6.99	7.65	8.15	8.70	9.22	9.94	10.40	11.07	11.86	12.98
$1-c_A c_B$	0.75	0.703	0.640	0.604	0.582	0.564	0.545	0.528	0.508	0.496	0.482	0.465	0.441
$a=0.25$—													
$c_A=c_B$	0.500	0.541	0.590	0.615	0.631	0.644	0.652	0.660	0.674	0.684	0.691	0.703	0.715
$\alpha=\beta$	4.00	4.71	5.77	6.43	6.60	7.31	7.61	7.94	8.45	8.80	9.20	9.73	10.35
$1-c_A c_B$	0.75	0.708	0.652	0.622	0.602	0.585	0.575	0.564	0.546	0.533	0.522	0.506	0.488
$a=0.20$—													
$c_A=c_B$	0.500	0.534	0.577	0.597	0.610	0.620	0.625	0.636	0.646	0.652	0.658	0.667	0.676
$\alpha=\beta$	4.00	4.56	5.39	5.87	6.20	6.46	6.64	6.92	7.26	7.44	7.64	7.94	8.38
$1-c_A c_B$	0.75	0.715	0.667	0.643	0.628	0.616	0.609	0.595	0.582	0.574	0.567	0.556	0.542
$a=0.15$—													
$c_A=c_B$	0.500	0.529	0.560	0.577	0.587	0.595	0.600	0.605	0.613	0.618	0.623	0.629	0.639
$\alpha=\beta$	4.00	4.44	5.01	5.34	5.55	5.73	5.85	5.99	6.17	6.31	6.45	6.60	6.88
$1-c_A c_B$	0.75	0.720	0.686	0.667	0.656	0.646	0.640	0.634	0.624	0.618	0.612	0.604	0.592
(*f*) Unsymmetrical, Parabolic Haunch													
$a=1.00$—													
c_A	0.500	0.451	0.394	0.364	0.346	0.331	0.320	0.307	0.290	0.280	0.269	0.256	0.238
c_B	0.500	0.581	0.714	0.796	0.864	0.918	0.970	1.030	1.112	1.162	1.233	1.333	1.485
α	4.00	5.51	8.55	10.92	13.10	15.00	16.75	19.25	22.85	25.50	29.00	34.70	44.60
β	4.00	4.30	4.71	5.00	5.24	5.40	5.55	5.74	5.96	6.15	6.36	6.66	7.15
$1-c_A c_B$	0.75	0.738	0.718	0.710	0.701	0.696	0.690	0.682	0.678	0.675	0.668	0.659	0.647
$a=0.50$—													
c_A	0.500	0.479	0.454	0.441	0.432	0.426	0.422	0.416	0.405	0.406	0.400	0.394	0.386
c_B	0.500	0.578	0.685	0.747	0.794	0.830	0.860	0.895	0.935	0.970	1.00	1.047	1.097
α	4.00	5.03	6.64	7.74	8.60	9.29	9.85	10.60	11.20	12.20	12.97	14.10	15.60
β	4.00	4.16	4.40	4.56	4.67	4.78	4.83	4.92	4.85	5.11	5.19	5.30	5.46
$1-c_A c_B$	0.75	0.724	0.690	0.670	0.656	0.646	0.637	0.627	0.621	0.606	0.600	0.588	0.576
$a=0.40$—													
c_A	0.500	0.486	0.468	0.460	0.454	0.450	0.445	0.441	0.436	0.435	0.430	0.426	0.421
c_B	0.500	0.567	0.659	0.710	0.746	0.775	0.794	0.822	0.856	0.874	0.895	0.929	0.970
α	4.00	4.84	6.15	6.95	7.52	8.02	8.40	8.86	9.54	9.90	10.40	11.02	11.90
β	4.00	4.15	4.36	4.49	4.58	4.65	4.71	4.78	4.85	4.92	4.99	5.05	5.17
$1-c_A c_B$	0.75	0.724	0.692	0.674	0.658	0.651	0.647	0.637	0.626	0.620	0.615	0.604	0.591
$a=0.35$—													
c_A	0.500	0.488	0.475	0.468	0.462	0.460	0.455	0.454	0.450	0.446	0.445	0.441	0.436
c_B	0.500	0.562	0.642	0.689	0.717	0.744	0.761	0.780	0.810	0.825	0.844	0.870	0.899
α	4.00	4.76	5.87	6.57	7.02	7.42	7.72	8.09	8.55	8.97	9.27	9.70	10.35
β	4.00	4.14	4.34	4.46	4.52	4.59	4.64	4.70	4.75	4.80	4.87	4.94	5.02
$1-c_A c_B$	0.75	0.726	0.695	0.678	0.668	0.658	0.655	0.646	0.636	0.632	0.624	0.616	0.608
$a=0.30$—													
c_A	0.500	0.490	0.481	0.475	0.470	0.468	0.466	0.465	0.462	0.460	0.457	0.455	0.451
c_B	0.500	0.555	0.625	0.663	0.689	0.710	0.725	0.744	0.762	0.775	0.793	0.810	0.835
α	4.00	4.66	5.60	6.15	6.53	6.83	7.08	7.35	7.71	7.92	8.23	8.56	9.00
β	4.00	4.12	4.30	4.40	4.46	4.51	4.55	4.60	4.67	4.70	4.74	4.81	4.86
$1-c_A c_B$	0.75	0.728	0.698	0.685	0.676	0.668	0.662	0.654	0.648	0.643	0.637	0.631	0.623
$a=0.25$—													
c_A	0.500	0.492	0.485	0.481	0.479	0.476	0.475	0.475	0.474	0.470	0.468	0.465	0.465
c_B	0.500	0.548	0.607	0.639	0.659	0.674	0.688	0.701	0.717	0.728	0.740	0.754	0.775
α	4.00	4.57	5.33	5.75	6.04	6.28	6.45	6.67	6.94	7.06	7.28	7.52	7.82
β	4.00	4.11	4.27	4.35	4.39	4.45	4.46	4.51	4.56	4.58	4.60	4.64	4.69
$1-c_A c_B$	0.75	0.730	0.706	0.692	0.685	0.679	0.673	0.667	0.660	0.658	0.654	0.650	0.640
$a=0.20$—													
c_A	0.500	0.495	0.490	0.488	0.486	0.486	0.484	0.484	0.481	0.481	0.479	0.479	0.476
c_B	0.500	0.540	0.587	0.613	0.629	0.639	0.650	0.660	0.674	0.680	0.688	0.701	0.716
α	4.00	4.45	5.06	5.38	5.57	5.75	5.88	6.05	6.21	6.32	6.45	6.64	6.85
β	4.00	4.10	4.21	4.27	4.31	4.36	4.39	4.43	4.45	4.47	4.49	4.53	4.56
$1-c_A c_B$	0.75	0.732	0.712	0.701	0.694	0.689	0.686	0.680	0.676	0.672	0.670	0.664	0.658
$a=0.15$—													
c_A	0.500	0.496	0.495	0.492	0.492	0.492	0.490	0.490	0.488	0.488	0.487	0.486	0.485
c_B	0.500	0.531	0.567	0.585	0.596	0.605	0.610	0.618	0.630	0.634	0.639	0.646	0.655
α	4.00	4.35	4.76	5.00	5.16	5.28	5.34	5.44	5.56	5.64	5.71	5.81	5.95
β	4.00	4.07	4.17	4.21	4.25	4.27	4.29	4.30	4.31	4.34	4.35	4.37	4.40
$1-c_A c_B$	0.75	0.736	0.719	0.712	0.706	0.702	0.701	0.697	0.693	0.691	0.688	0.686	0.682

fixed against rotation. Also, the fixed-end moment at end A, which is induced by angular displacement R, is equal to $-S_A\ (1 - c_A\ c_B)\ R$.

Six tables giving carry-over factors and stiffness coefficients, and also values of the factor $(1 - c_A\ c_B)$, for members having varying moment of inertia have been prepared from the tables compiled by Mr. Ruppel (21) and are given in Tables 6(a) to 6(f), inclusive.

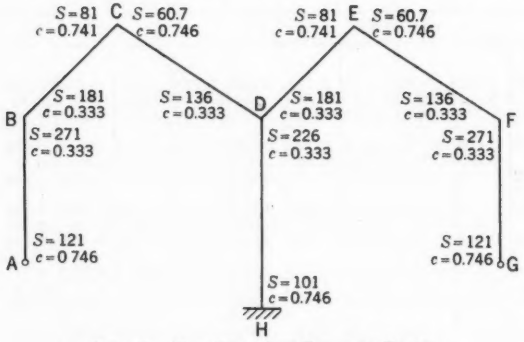


FIG. 35.—TWO-BAY, SAW-TOOTHED FRAME

Special Solution by Slope Deflection.—A special technique in the use of slope deflection, similar to that discussed previously herein for frames having members of constant section, will be applied to the frame analyzed by Mr. McGee. For the purpose of determining the effects of an applied moment at any joint of a one-story frame or continuous beam, the general slope-deflection equation for members having varying moments of inertia may be written conveniently in the following form:

$$M\ (\text{near}) = S\ (\text{near})\ [\theta\ (\text{near}) + c\ (\text{near})\ \theta\ (\text{far})] \ldots\ldots\ldots (98)$$

Fig. 35 represents the frame analyzed by Mr. McGee with relative values of the stiffnesses and values of the carry-over factors (taken from Table 6(*d*)) recorded at the ends of the appropriate members. It was first assumed that $\theta_G = 0.427$, in order to show the relation between the slope-deflection and traverse procedures (ordinarily θ_G would be assumed equal to unity).

By applying Eq. 98 successively to the several members of the frame, the following simple calculations are made. First assume $\theta_G = 0.427$; then:

$$M_{GF} = 121\ (0.427 + 0.746\ \theta_F) = 0; \quad \theta_F = -0.573$$
$$M_{FG} = 271\ [-0.573 + (0.333)\ (0.427)] = -117$$

$$M_{FE} = 136\ [-0.573 + 0.333\ \theta_E] = 117; \quad \theta_E = 4.30$$
$$M_{EF} = 60.7\ [4.30 + (0.746)\ (-0.573)] = 236$$

$$M_{ED} = 81\ [4.30 + 0.746\ \theta_D] = -236; \quad \theta_D = -9.66$$
$$M_{DE} = 181\ [-9.66 + (0.333)\ (4.30)] = -1,490$$

$$M_{DH} = 226\ [-9.66 - 0] = -2,180$$
$$M_{HD} = 101\ [0 + (0.746)\ (-9.66)] = -730$$

$$M_{DC} = 136\ [-9.66 + 0.333\ \theta_C] = 3,670; \quad \theta_C = 110$$
$$M_{CD} = 60.7\ [110 + (0.746)\ (-9.66)] = 6,240$$

$$M_{CB} = 81\ [110 + 0.746\ \theta_B] = -6,240; \quad \theta_B = -251$$
$$M_{BC} = 181\ [-251 + (0.333)\ (110)] = -38,700$$

$$M_{AB} = 121\ [\theta_A + (0.746)\ (-251)] = 0; \quad \theta_A = 187$$
$$M_{BA} = 271\ [-251 + (0.333)\ (187)] = -51,100$$

The foregoing moment values are shown at the ends of the members in Fig. 36(*a*). It is obvious that an applied moment of −89,800 ft-lb at joint B will induce these moments throughout the frame, and an angular rotation at joint G equal to 0.427 (which was assumed). It will be observed that the values of θ obtained in the foregoing solution check those in Fig. 21, and that the calculated moments are proportional to the values shown in the same figure.

Similarly, if one starts at joint B and assumes $\theta_B = 0.427$, the moment values shown in Fig. 36(*b*) are obtained. It will be observed that the moment at joint G required to produce this angular rotation is also equal to −89,800 ft-lb. This constitutes a valuable check on the computations, in agreement with Maxwell's law of reciprocity of displacement. From the moment values in Figs. 36(*a*) and 36(*b*), the effects of applied moments at joints C, D, E, and F can be determined easily in a manner exactly similar to that shown for frames having members of constant section (see Fig. 33).

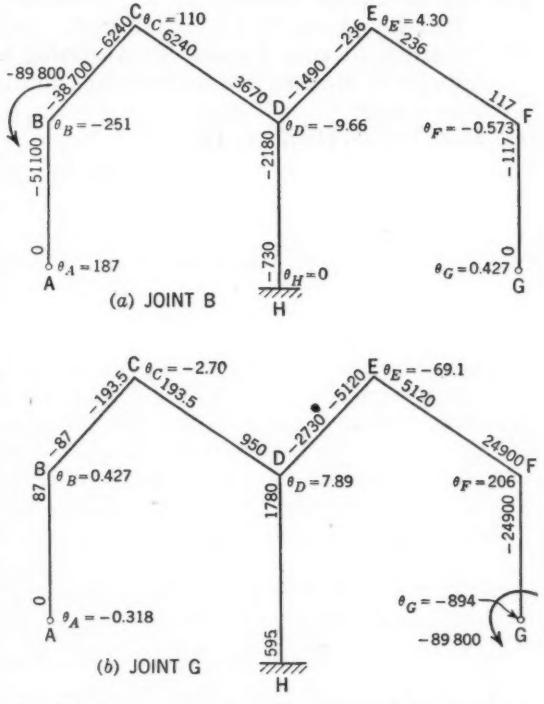


FIG. 36.—EFFECTS OF APPLIED MOMENTS AT DIFFERENT JOINTS

Summary.—It has been demonstrated in the paper, and also in the discussions, that the principle of superposition is indispensable in the analysis of rigid frames by the slope-deflection and moment distribution methods where applied loads cause linear displacements of the joints.

As stated in the paper, the key to analysis in any case consists in the consideration of all possible conditions of deformation of the given frame. This initial step of the method of procedure, which is based upon the validity of superposition, is very important and cannot be overemphasized. The balancing of fixed-end moments at the joints by the general methods of slope deflection or moment distribution (or by special methods where they apply) is but one step in the solution of a rigid frame problem involving unknown linear displacements of the joints, as the outline of the method of procedure clearly shows. The principle of superposition, on the other hand, is applied constantly throughout the analysis and, therefore, is deserving of special emphasis.

Bibliography.—

(3) **"Continuous Frames of Reinforced Concrete,"** by Hardy Cross and N. D. Morgan, John Wiley & Sons, Inc., New York, N. Y., 1932. (*a*) p. 209. (*b*) p. 110. (*c*) p. 113.

(11) **"Elastic Energy Theory,"** by J. A. Van den Broek, John Wiley & Sons, Inc., New York, N. Y., 2d Ed., 1942. (*a*) pp. 226–229.

(17) **"Relative Flexure Factors for Analyzing Continuous Structures,"** by Ralph W. Stewart, *Transactions*, Am. Soc. C. E., Vol. 104 (1939), p. 521. (*a*) p. 542.

(21) *Ibid.*, Vol. 90 (1927), p. 170.

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DISCUSSIONS

THE EFFECT OF BLANKETS ON SEEPAGE THROUGH PERVIOUS FOUNDATIONS

Discussion

BY REGINALD A. BARRON

REGINALD A. BARRON,[3] JUN. AM. SOC. C. E.[3a]—The efficiency of upstream blankets for dams and dikes has been presented in an interesting manner by Mr. Bennett. The writer agrees with the author as to the effects of variations in extent and physical properties of natural soil deposits, and believes that the simplifying assumptions made in the paper are quite acceptable. The difference in permeability between natural blanket and underlying pervious layer quite easily may be greater than 1 to 10 when the effect of stratification is considered.

The writer has developed and used comparable equations particularly for the study of hydraulic pressures on the lower side of a natural blanket for condi-

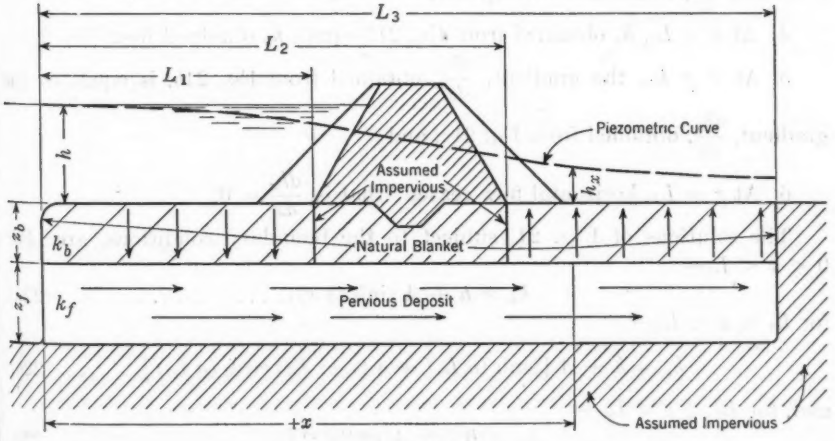

FIG. 6.

NOTE.—This paper by Preston T. Bennett was published in January, 1945, *Proceedings*.

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[3a] Received by the Secretary February 13, 1945.

tions shown in Fig. 6. In this case an earth embankment with a central impervious core and flanking pervious shells rests upon a natural blanket which in turn rests upon a more pervious deposit in direct contact with the upstream pool. The assumptions are quite similar to those of the author. In the length L_1, it is assumed that the flow is vertically downward in the blanket. In the length $L_2 - L_1$, seepage through the embankment and the underlying blanket is neglected. In the remaining length $L_3 - L_2$, the seepage through the blanket is vertically upward. In the pervious deposit the seepage is considered to occur in a horizontal direction only.

The differential equations based upon continuity of flow are: For $0 < x < L_1$,

$$-k_f \frac{d^2h_x}{dx^2} \cdot z_f = +k_b \frac{h - h_x}{z_b} \quad \ldots\ldots (21a)$$

for $L_1 < x < L_2$,

$$-k_f \frac{d^2h_x}{dx^2} \cdot z_f = 0 \quad \ldots\ldots (21b)$$

and, for $L_2 < x < L_3$,

$$+k_f \frac{d^2h_x}{dx^2} \cdot z_f = +k_b \frac{h_x}{z_b} \quad \ldots\ldots (21c)$$

in which h_x is the elevation of the piezometric height above land-side tailwater.

The boundary conditions are:

1. At $x = 0$, $h_x = h$;
2. At $x = L_1$, h_x obtained from Eq. 21*a* equals h_x obtained from Eq. 21*b*;
3. At $x = L_1$, the gradient, $\frac{dh_x}{dx}$, obtained from Eq. 21*a*, is equal to the gradient, $\frac{dh_x}{dx}$, obtained from Eq. 21*b*;
4. At $x = L_2$, h_x obtained from Eq. 21*b* equals h_x obtained from Eq. 21*c*;
5. At $x = L_2$, the gradient, $\frac{dh_x}{dx}$, obtained from Eq. 21*b*, is equal to the gradient, $\frac{dh_x}{dx}$, obtained from Eq. 21*c*; and
6. At $x = L_3$, horizontal flow ceases—that is, $\frac{dh_x}{dx} = 0$.

The solutions of Eqs. 21, subject to the boundary conditions, are: For $0 < x < L_1$—

$$h_x = h + A \sinh (a\,x) \quad \ldots\ldots (22a)$$

for $L_1 < x < L_2$—

$$h_x = h + A\,[\sinh (a\,L_1) + a\,(x - L_1) \cosh (a\,L_1)] \quad \ldots\ldots (22b)$$

and, for $L_2 < x < L_3$—

$$h_x = B\,[e^{ax} + e^{a(2L_3 - x)}] \quad \ldots\ldots (22c)$$

in which

$$A = B \left\{ \frac{[e^{aL_2} - e^{a(2L_3 - L_2)}]}{\cosh (a\,L_1)} \right\} \quad \ldots\ldots (23a)$$

$$B = h \div \{[1 - \tanh(a L_1) - a(L_2 - L_1)]\, e^{aL_1} + [1 + \tanh(a L_1) + a(L_2 - L_1)]\, e^{a(2L_3 - L_2)}\} \quad \ldots\ldots (23b)$$

and

$$a = \sqrt{\frac{k_b}{k_f z_b z_f}} \quad \ldots\ldots (23c)$$

The seepage rate per unit width at any section in the pervious deposit may then be obtained from:

$$Q_x = -k_f \frac{dh_x}{dx} z_f \quad \ldots\ldots (24a)$$

and the seepage rate per unit width at any point x through the downstream natural blanket may be obtained from

$$Q'_x = k_b \frac{h_x}{z_b} \quad \ldots\ldots (24b)$$

If the dimension L_3 approaches infinity, Eqs. 22 become: For $0 < x < L_1$—

$$h_x = h \left\{ 1 - \frac{\sinh(a x)}{[1 + \tanh(a L_1) + a(L_2 - L_1)] \cosh(a L_1)} \right\} \quad \ldots (25a)$$

for $L_1 < x < L_2$—

$$h_x = h \left[\frac{1 + a(L_2 - x)}{1 + \tanh(a L_1) + a(L_2 - L_1)} \right] \quad \ldots\ldots (25b)$$

and, for $L_2 < x$—

$$h_x = \frac{h\, e^{a(L_2 - x)}}{1 + \tanh(a L_1) + a(L_2 - L_1)} \quad \ldots\ldots (25c)$$

If the distance L_1 approaches minus infinity and L_3 approaches plus infinity, then, with the origin at the upstream toe of the impervious core of the earth embankment and with $(L_2 - L_1)$ equal to L, Eqs. 25 further simplify as follows: For $x < 0$—

$$h_x = h \left[1 - \frac{e^{ax}}{2 + a L} \right] \quad \ldots\ldots (26a)$$

for $0 < x < L$—

$$h_x = h \left[\frac{1 + a(L - x)}{2 + a L} \right] \quad \ldots\ldots (26b)$$

and, for $L < x$—

$$h_x = \frac{h\, e^{a(L - x)}}{2 + a L} \quad \ldots\ldots (26c)$$

Eqs. 25*a* and 26*a* are in agreement with Eqs. 6 and 9*a*, respectively, when proper consideration is given to the physical constants and boundaries.

The foregoing development neglects the fact that the seepage paths are curved lines in the pervious deposit. Because of this phenomenon, the piezometric heads along the upper part of the pervious deposit will be somewhat less at the river-side toe of the embankment and somewhat greater at the land-side toe, as compared with the piezometric heads obtained from flow-net studies.

Since it is possible that the land-side natural blanket may be disrupted by excess water pressure from pervious deposit, this difference in calculated heads is conservative. The neglect of seepage through the embankment and through the natural blanket immediately below reduces the seepage calculated by Eq. 24*a* as compared with that calculated by flow-net studies. The reduction is not large and is well within the permeability range of natural deposits.

Although the author indicates that the law of diminishing returns applies to an upstream blanket extended beyond certain limiting lengths, the writer would be somewhat reluctant to destroy or weaken any such natural blanket because of the increased water pressures that would occur under the blanket in the downstream area. If upstream borrow pits are used, adequate drainage facilities should be provided at the downstream toe or at adjacent areas to minimize the danger from increased water pressures. Downstream borrow pits, if available, would appear to be more desirable, and might aid in partly controlling any excessive pressures under the blanket between the downstream embankment toe and the borrow pits. However, borrow pits should be excavated so as to minimize the danger from piping or underground erosion which might work back under the natural blanket to the upstream area.

An upstream blanket with variable section is not a new idea but the author has done well to restate the case and to indicate the proper and most beneficial use of available material. A review of a number of dams indicates that blankets with triangular sections have been utilized infrequently. However, triangular-shaped blankets have certain practical limits that should be considered. For instance, it is doubtful that a blanket shaped to a featheredge would be either desirable or practical to construct. For flood control structures that do not have permanent storage, partial loss of imperviousness may occur in an exposed upstream blanket near the surface due to loosening of the blanket by frost action or due to other causes which promote soil expansion.